Distributed input-delay Model Predictive Control of inland waterways

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Abstract

Inland waterways are large, complex systems composed of interconnected navigation reaches dedicated mainly to navigation. These reaches are generally characterized by negligible bottom slopes and large time delays. The latter requires ensuring the coordination of the current control actions and their delayed effects in the network. Centralized control strategies are often impractical to implement due to the size of the system. To overcome this issue, a distributed Model Predictive Control (MPC) approach is proposed. The system partitioning is based on a reordering of the optimality conditions matrix, and the control actions are coordinated by means of the Optimality Condition Decomposition (OCD) methodology. The case study is inspired by a real inland waterways system and shows the performance of the approach.

1 Introduction

Inland navigation networks are large-scale systems, often regarded as the interconnection of several reaches. The management of these systems aims at keeping the water levels close to the setpoints, a condition that must be met to ensure the accommodation of navigation. Since the water levels are controlled by means of gates located at the junctions of the reaches, the control laws must take into account the network configuration. Otherwise, the local control objective for one reach might be achieved at the expense of the control objectives of other reaches. Centralized control approaches for these systems are often impractical and can lead to implementation problems due to the spatial distribution and multi-time scales [1]. To overcome these issues, decentralized control approaches were conceived: the system is partitioned into subsystems, and a local controller is in charge of meeting the control objective for each subsystem. Such strategies usually solve these sub-problems by considering other subsystems’ inputs as external disturbances, which might lead to a poor overall performance [2]. However, the exchange of information among local controllers is possible nowadays thanks to the developments in information and communications technology, which allow these controllers to cooperate and negotiate with each other, aiming at achieving the best global performance [3]. Architectures in which these communication protocols between subsystems are implemented are referred to as distributed control techniques.
Certain aspects of the management of modern water systems require advanced control methods [4]. For this reason, Model Predictive Control (MPC) is applied in the present work. This methodology is very well suited in those cases where the future references are known. Roughly speaking, reference values are the desired water levels in the reaches, while lock operations that allow boats to cross from one reach to the next one are regarded as disturbances. Gates are used to dispatch water along the system so that the control objectives are satisfied and the disturbances are rejected.

The framework of decentralized and distributed predictive control of water systems has attracted considerable interest in the past years. A decentralized adaptive predictive controller for irrigation canals was designed in [5], aiming at controlling the downstream water levels of a set of reaches. An optimal decentralized control architecture was presented in [6] to ensure the efficient management of an inland navigation network in a global change context. Another decentralized controller was designed in [7] for a part of a real inland navigation network with a distributary. The application of non-centralized approaches to drinking water networks to improve the performance with respect to the centralized counterpart was discussed in [8]. A comparison of decentralized and distributed control strategies for irrigation systems was performed in [9], where the benefits of cooperative distributed control were validated. A hierarchical distributed MPC approach applied to irrigation canal planning was presented in [10], addressing a risk management strategy.

Summary of the paper and contributions

The centralized MPC scheme presented in [11] might not be practical to implement in the case of large-scale systems due to the spatial distribution of the network. To overcome this limitation, this work proposes a distributed MPC approach based on the initial centralized design. The formulation and manipulation of the Karush-Kuhn-Tucker (KKT) centralized matrix yields separable KKT subsystems, which define the structure of the decomposed control sub-problems [12]. Once this decomposed structure is attained, the Optimality Condition Decomposition (OCD) technique [13] is used to obtain the consensus strategy that leads to the best global performance. The rest of the paper is organized as follows: Section 2 summarizes the centralized MPC formulation presented in the aforementioned work. In Section 3, the centralized KKT system is formulated, and it is shown how to manipulate it to obtain the distributed KKT system. Furthermore, the OCD technique is presented, and the final distributed MPC formulation is given. An illustrative case study, inspired by part of a real inland waterways network, is presented in Section 4, which illustrates the proposed approach and highlights the performance of the control strategy. Section 5 draws conclusions and outlines future steps.

Notation

Throughout this paper, let \( \mathbb{R}^n \) denote the set of column real vectors of length \( n \). Scalars are denoted with either lowercase or uppercase letters (e.g. \( \alpha, a, A, \) etc.); vectors, with bold lowercase letters (e.g. \( \mathbf{a}, \mathbf{b}, \) etc.); and matrices, with bold uppercase letters (e.g. \( \mathbf{A}, \mathbf{B}, \) etc.). Furthermore, all vectors are column vectors unless otherwise stated, and \( \mathbf{0} \) denotes a zero column vector of suitable dimensions. Transposition is denoted with the superscript \(^\top\), and the operators \(<, \leq, =, \geq \) and \( > \) denote element-wise relations of vectors.

2 Centralized MPC formulation

Due to lack of space, only the main features of the centralized MPC formulation presented in [11] are summarized in this section. More details about its development can be found therein.
2.1 IDZ model and equivalent state-space formulation

In order to develop an MPC for inland waterways, a model that describes the dynamic behavior of the system is needed. The Saint-Venant (SV) differential equations can accurately describe the real dynamics of the system [14]. However, these equations are not well suited for control purposes as they have no known analytical solution and are very sensitive to errors in the parameters. Many simplified models have been proposed to deal with these issues; among all of them, the Integral Delay Zero (IDZ) model [15] is used in this work. The general IDZ input-output expression that links the discharges and the water depths at the boundaries of a reach is given by:

\[
\begin{bmatrix}
    y(0,s) \\
    y(L,s)
\end{bmatrix} = \begin{bmatrix}
    p_{11}(s) & p_{12}(s) \\
    p_{21}(s) & p_{22}(s)
\end{bmatrix} \begin{bmatrix}
    q(0,s) \\
    q(L,s)
\end{bmatrix},
\]

where 0 and L are the abscissas for the initial and final ends of the canal; \( y(0,s) \) and \( y(L,s) \), the upstream and downstream water levels; \( q(0,s) \) and \( q(L,s) \), the upstream inflow and downstream outflow; and \( p_{1j}(s) = \frac{\alpha_j \cdot e^{-\tau_j s}}{A_{ij}} \), the different terms of the IDZ model (\( A_{ij} \) is the integrator gain, \( \tau_j \) is the time delay and \( \alpha_j \) is the inverse of the zero). The parameters of the first equation of (1) are linked to the upstream water level, while those in the second equation are linked to the downstream water level. Based on this, the notation of the parameters is modified in [15] and is adopted in the present work: \( A_{11} = A_{12} = A_u, A_{21} = A_{22} = A_d, \tau_{12} = \tau_r \) and \( \tau_{21} = \tau_d \) (\( \tau_{11} = \tau_{22} = 0 \)).

An equivalent discrete state-space formulation is obtained to ensure the correct coordination between actual control values and their delayed effect in the system (a sampling period \( T_s \) is used):

\[
x_{k+1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} T_s & 0 \\ 0 & T_s \end{bmatrix} u_k + \begin{bmatrix} 0 & T_s \\ T_s & 0 \end{bmatrix} u_{k-n} ; \quad y_k = \frac{1}{A_u} \begin{bmatrix} 1 & 0 \end{bmatrix} x_k + \frac{\alpha_{11}}{A_u} \begin{bmatrix} 0 & 1 \end{bmatrix} u_k + \begin{bmatrix} 0 & \alpha_{21} \end{bmatrix} u_{k-n}
\]

(2)

with \( k \) the discrete-time instant and \( u_{k-n} \) the input vector delayed \( n \) samples (\( n = \lceil \tau/T_s \rceil \), with \( \lceil \cdot \rceil \) the ceiling function). In practice, the numerical values of \( \tau_d \) and \( \tau_r \) are almost the same, leading to a single value of \( n \). Equation (2) describes a reach with two inputs and two outputs. The formulation of a system with \( n_x \) states, \( n_u \) inputs and \( n_y \) outputs, and with demands \( d_k \) (acting as additive disturbances), is:

\[
x_{k+1} = A x_k + B_u u_k + B_{u-n} u_{k-n} + B_d d_k + B_{d-n} d_{k-n} ; \quad y_k = C x_k + D_u u_k + D_{u-n} u_{k-n} + D_d d_k + D_{d-n} d_{k-n}
\]

(3)

2.2 Control design

The main principle of MPC techniques resides in computing a control sequence that makes the predicted response move to the setpoint in an optimal manner without violating the constraints. Thus, it is necessary to define the constraints and the operational goals to be fulfilled. The system functioning is constrained by the physical nature of the variables as well as some elements in the waterways. Each of the constraints is described and formulated below:

- Mass balance relations must be imposed at the nodes: \( 0 = E_{u} u_k + E_{d} d_k \).
- The lower and upper bounds of the \( m \)-th actuator must be respected: \( u^m_k \leq u^m_k \leq u^m_k \), \( m = 1, \ldots, N_m \), with \( N_m \) the total number of actuators in the system.
- The water levels \( y_k \) must be kept within the predefined navigation interval \([y_L, y_U]\) to ensure the navigability: \( y_L - \alpha_k \leq y_k \leq y_U + \alpha_k \).
Furthermore, the several operational goals that are to be fulfilled are the following:

- Maintain the water levels close to the setpoints \( y_r \): 
  \[ J^1_k = (y_k - y_r)^T (y_k - y_r) \]

- Reduce the economic cost derived from the operation of the available controlled equipment: 
  \[ J^2_k = \gamma u_k^T u_k \]

- Guarantee a smooth control action: 
  \[ J^3_k = \Delta u_k^T \Delta u_k \]

- Penalize the relaxation of the navigability condition to be as little as possible outside the navigation interval: 
  \[ J^4_k = \alpha \alpha^T \alpha_k \]

The objectives are gathered to build the objective function 
\[ J = \sum_{k=1}^{H_p} \sum_{j=1}^{4} \beta^j J^j_k, \]
where \( H_p \) is the prediction horizon, and the weights \( \beta^j \) are selected as shown in [11]. The solution of the centralized control problem is then given by:

\[
\begin{align*}
\min J & \quad \text{(4a)} \\
\text{subject to:} & \\
& X_{k+1} |_{k+i} = AX_{k+i} |_{k} + Bu_{k+i} |_{k} + Bu_{k+i-1} |_{k} + Bu_{k+i-2} |_{k} + Bu_{k+i-3} |_{k} + Bu_{k+i-4} |_{k} \\
& Y_{k+i} |_{k} = CX_{k+i} |_{k} + Du_{k+i} |_{k} + Du_{k+i-1} |_{k} + Du_{k+i-2} |_{k} + Du_{k+i-3} |_{k} + Du_{k+i-4} |_{k} \\
& 0 = Eu_{k+i} |_{k} + Du_{k+i} |_{k} \\
& u^m \leq u_k \leq u^u \quad \text{(4e)} \\
& y_r - \alpha_k \leq y_k \leq y_r + \alpha_k \quad \text{(4f)} \\
& \alpha_k \geq 0 \quad \text{(4g)}
\end{align*}
\]

with \( H_p \) the prediction horizon, \( i \) the time instant along the prediction horizon, \( k \) the current time instant and \( k+i |_{k} \) the time instant \( k+i \) given \( k \).

## 3 Distributed MPC formulation

As it has been stated before, centralized control approaches for large-scale systems are often difficult to implement, mainly due to the spatial distribution of the elements in the network. To overcome this limitation, the system is partitioned into sub-systems. In this work, this decomposition step is carried out by manipulating the centralized Karush-Kuhn-Tucker (KKT) matrix of the large-scale problem, which yields separable KKT subsystems that define the structure of the decomposed sub-problems. Once such a decomposed problem structure is obtained, the Optimality Condition Decomposition (OCD) technique is used to coordinate the local controllers to guarantee the best global performance.

### 3.1 Centralized and distributed KKT systems

To formulate the centralized KKT system, it is necessary first to define the Lagrangian function

\[
L(u, y, \lambda_1, \lambda_2, \lambda_3) = f(u, y) + \sum_{j=1}^{H_p} \left( \lambda_1 h^{(1)}_j(u, y) + \lambda_2 h^{(2)}_j(u, y) + \lambda_3 h^{(3)}_j(u, y) \right),
\]

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where $f(u, y)$ is the objective function given in (4a); $\lambda_1$, $\lambda_2$ and $\lambda_3$ are the Lagrange multipliers; and $h_j^{(1)}$, $h_j^{(2)}$ and $h_j^{(3)}$ are the three first constraints of the optimization problem (4), respectively. Indeed, the bounds on the variables do not affect the decomposition, and hence they are not considered in this step. Note that the temporal dependence of $u$ and $y$ is not explicitly indicated for readability.

The KKT matrix for the overall system is built using (5), which can be obtained by applying the primal dual interior point method or the gradient-based method [16]. This matrix represents the optimality conditions and is denoted with $\text{KKT}_{\text{cent}}$. The next step consists in manipulating this matrix such that a block-diagonal structure is attained. A number of methods can be employed to transform a symmetric matrix such as $\text{KKT}_{\text{cent}}$ into the block-diagonal form. However, some of them are not well suited for the large $\text{KKT}_{\text{cent}}$ matrices that result from the MPC problem, while others compromise the coupling information due to the elimination of some matrix coefficients. The Cuthill-McKee ordering algorithm [17] performs row/column permutation operations in symmetric matrices to obtain a block-diagonal matrix with minimal couplings. This reordering provides as a solution $l$ KKT block matrices on the diagonal, which can be regarded as $l$ subsystems into which the overall system can be decomposed. The final block-diagonal matrix is denoted with $\text{KKT}_{\text{dist}}$.

### 3.2 Optimality Condition Decomposition

Once the system partitioning has been performed, the local controllers must be designed, and their actions coordinated. The OCD technique decomposes the centralized problem into $l$ sub-problems, and the constraints are decomposed into $l$ groups of constraints, each of them describing the dynamics of a subsystem and the interactions with the rest of subsystems. At each iteration, the method fixes, for the $i$-th sub-problem, all variables to their last computed values (denoted in (6) with a tilde), except for the $i$-th group of variables. The $l$ parallel sub-problems are given by

$$
\min_{u, y, j} \left\{ f(u_1, y_1, \ldots, u_i, y_i, \ldots, u_l, y_l) + \sum_{j=1, j \neq i}^l \lambda_j h_j(u_1, y_1, \ldots, u_i, y_i, \ldots, u_l, y_l) \right\}
$$

subject to:

$$
h_j(u_1, y_1, \ldots, u_i, y_i, \ldots, u_l, y_l) = 0; \quad u_j^m \leq u_j \leq u_j^M; \quad y_j - \alpha_j \leq y_j \leq y_j + \alpha_j; \quad \alpha_j \geq 0
$$

Once the solution has been computed for each sub-problem, the Lagrangian multipliers are updated, following, for instance, a sub-gradient technique: $\lambda_j^{(v+1)} = \lambda_j^{(v)} + \chi h_j$, with $v$ the current iteration and $\chi \in (0, 1)$ a suitable constant. The coordination of the global problem is achieved through the Lagrange multipliers. On the other hand, the convergence speed is characterized by the accepted variation of the values of the multipliers between consecutive iterations.

### 4 Case study

The result of applying the distributed MPC given by the solution of (6) is presented in this section. The system is first described, then the experimental design is presented, and finally the results are shown and the controller performance is discussed.

#### 4.1 System description

A system based on the inland navigation network in the north of France is used to illustrate the modeling and control techniques. It is schematized in Fig. 1. N1, N2, N3 and N4 are equipped with controlled
gates that allow dispatching water to fulfill the control objectives, as well as locks used by boats to cross from one reach to the adjacent one. Bif stands for bifurcation, where a gate is used to regulate the flow supplied to reaches 2 and 3. Four identical reaches with the following magnitudes make up the system: $L = 27000$ m (length of the reach), $w_r = 50$ m (bottom width), $m_r = 0$ (side slope of the reach, $m_r = 0$ for a rectangular cross section), $s_b = 0$ (bottom slope, $s_b = 0$ for a flat reach), $n_r = 0.035$ $m^{-1/3}$ (Manning roughness coefficient) and $Q_s = 0.6$ $m^3 s^{-1}$ (operating point considered in the SV equations linearization). The navigability condition is ensured if the water levels are kept in the interval $3.8 \pm 0.1$ m. However, this objective is disturbed by lock operations with magnitudes: $18000$ $m^3$ (N1), $18000$ $m^3$ (N2), $9000$ $m^3$ (N3) and $24000$ $m^3$ (N4), with an average duration of $20$ min in all cases. In each operation, the corresponding water volume is withdrawn from the upstream reach and released into the downstream reach. Thus, the sign of these uncontrolled discharges will depend on whether these nodes are upstream or downstream nodes, i.e. on the system partitioning. In addition, the lock operation time-series model is considered to be known in advance. Indeed, a common waterways management policy dictates that, when a boat passes through a lock, its manager informs the manager of the next lock so that the arrival time of the boat can be anticipated.

4.2 Experimental design

First, the centralized representation (4) is computed for the system depicted in Fig. 1. A sampling time $T_s = 20$ min is considered. The bounds on the gates are $\pm 60$ $m^3 s^{-1}$. Next, the decomposition techniques presented in Section 3.1 are applied to the centralized model. As a result, $KKT_{dist}$ is formed by two blocks, which indicates that the overall system can be decomposed into two subsystems. The first subsystem (SS.1) comprises reaches 1, 2 and 3, while the second one (SS.2) is only formed by reach 4. The only existing coupling is the input at node N2 (all the water withdrawn from reach 2 is released in reach 4). There is no coupling in the state at N2 as the gate allows different upstream and downstream water depths. Due to lack of space, neither $KKT_{cen}$ nor $KKT_{dist}$ are presented. A 24-hour navigation period is simulated to show the performance of the approach. Two different periods are distinguished: navigation (from 6 a.m. to 8 p.m.) and stoppage (from 8 p.m. to 6 a.m.). During the stoppage period, the boats are not allowed to navigate along the waterways. Figure 2 depicts the simulated lock operation time-series profile.
4.3 Results

The presented scenario is simulated, and the set of controlled actions and the predicted water levels are depicted in Fig. 3 and 4, respectively. Since there exists a proportionality between states and outputs (defined by matrix $C$), the states are not depicted. Note also that there are three variables for $B_i f$ and two for $N_2$: the distinction is made by adding the reach number after the variable name.

The simulation starts at 5 a.m., one hour before the navigation period. After this period ends (during which the system is disturbed), the water levels return to their equilibrium values. Figure 4 shows that the distributed MPC is able to keep the levels inside the navigation interval in the presence of disturbances. Note that the absolute values of the control signals in Fig. 3 are exactly the same for the two inputs at node $N_2$, where an input coupling exists (different sign due to the sign criterion). Furthermore, the set of control actions is kept within the equipment design range. However, one of the operational goals consisted in guaranteeing a smooth control signal by minimizing $\Delta u_k$. Although the control signals do not exhibit the smoothest behavior, the differences between two consecutive control actions are not so large compared to the operational range. This fact should result in a long lifespan of the equipment.

To quantify the controller performance, consider the tracking indices (7) defined as the error between the predicted levels $y_k$ and the setpoints $y_r$, with $\frac{1}{\tau} \left( y_r - y_r \right)$ the maximum deviation from $y_r$. The
lowest TE index is 97.7%, showing that this approach provides satisfactory results and that the tracking performance is guaranteed (TE=100% corresponds to the perfect tracking performance).

\[
TE[\%] = 100 \times \left[ 1 - \frac{1}{H_p} \sum_{k=1}^{H_p} \left( \frac{y_k - y_r}{y_r} - \frac{1}{2} \left( \frac{y_r - y_f}{y_f} \right) \right)^2 \right]
\]

5 Conclusions

This work proposed a distributed MPC approach to keep the levels of inland waterways within the limits in the presence of disturbances created by lock operations. The original centralized problem was presented and divided in sub-problems by manipulating the KKT centralized matrix. As a result, separate sub-problems were obtained, and each of them was taken care of by a local controller. The OCD technique was used to coordinate the controllers to obtain the best global performance. This approach proved to perform well, as it was shown by means of the case study. Thus, it can be stated that this methodology constitutes an efficient approach for large-scale systems, for which the centralized counterparts are difficult to implement due to the spatial distribution of the elements in the network.

In future works, the effect of possible sensor and actuator faults will be addressed. Therefore, fault-tolerant control techniques using the presented distributed MPC approach will be considered.

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References


