Bang Bang Spectrum Sensing

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Abstract

In this research paper, given an upper bound on total number of sensing bands, the problem of determining number of sensing bands with low and high sensing times (given the total sensing time as well as traffic over wideband of channels) is formulated and solved. In such formulation, the optimization criterion is chosen to be (maximization/minimization) average number of sensing bands. The dual problem deals with optimization (maximization/minimization) of low and high sensing times. The solution of dual problem readily follows. Hence, a bang bang time optimal spectrum sensing related results follow readily. It is shown that the variance of a Bernoulli Random variable, Z constitutes a logistic map in success probability 'P'. This fact is utilized to derive analytic results. Some analytical results related to number of sensing bands required are proved. Specifically it is shown that when the values assumed by Z are known integers, there is no value of success probability 'P' for which expectation of Z equals the variance of Z.

Index Terms

Spectrum Sensing, Cognitive Radio, Independent Identically Distributed Random Process, Source Coding

I. INTRODUCTION

Cognitive Radio has led to the concept of efficient utilization of electromagnetic spectrum [1], [2]. Particularly interweaving paradigm(of Cognitive Radio) requires spectrum sensing by

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the secondary users. When spectrum sensing has to be performed in real time, over large band of frequencies. efficient utilization of available sensing time is crucial [2], [28], [29], [30]. Thus, there are research efforts to utilize the historical traffic in spectrum bands to optimize the spectrum sensing procedures [31], [32], [34], [4], [37], [7]. Specifically in [9], wideband time optimal spectrum sensing based on historical traffic is discussed. This research paper is a continuation of that effort.

The idea of taking historical traffic (in-various bands) into account in spectrum sensing has been proposed by various researches [4], [9]. The specific approach proposed in this research paper is motivated by time-optimal control (Bang Bang control) in control theory [38]. Also, other motivating ideas for this research work are Water Pouring idea in information theory, source coding of binary alphabet source [8]. The research effort in this paper has its origins in the earlier work [9].

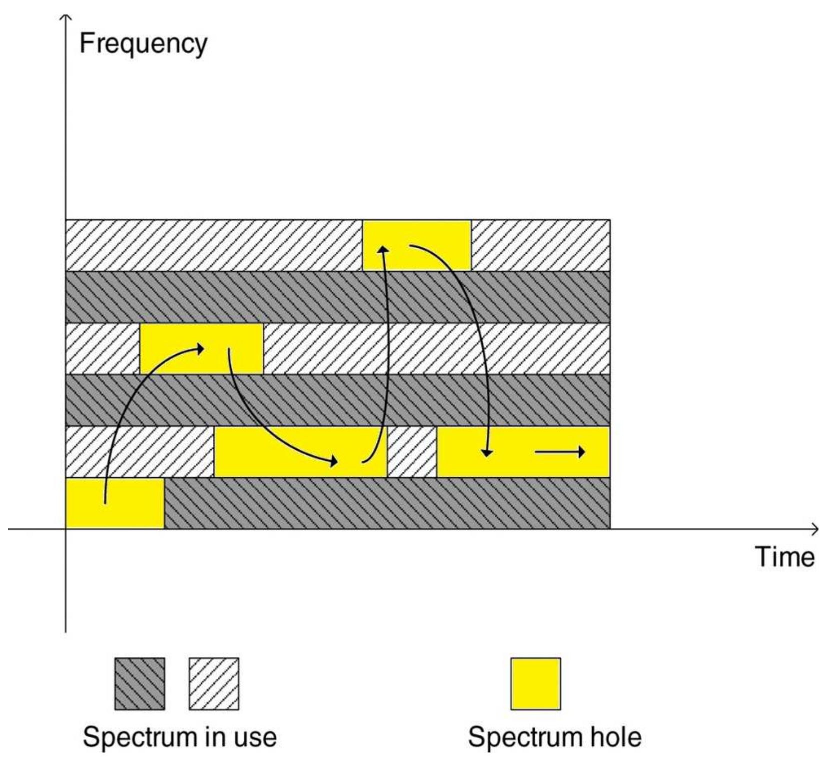

Fig. 1. Spectrum Occupancy

This paper is organized as follows. In section II, the problem of time optimal spectrum sensing using High, Low sensing times (in various bands) is formulated and solved. The results in section II are modified in section III. In section IV, numerical results are included. Finally conclusions are reported in section V.

II. HIGH/LOW SPECTRUM SENSING TIMES:OPTIMAL SPECTRUM SENSING

Spectrum sensing by secondary users is necessary to implement the ”interweaving” paradigm of Cognitive Radio. Researchers have proposed various spectrum sensing methods such as the energy detection based methods [6], [10], [11], [12], [13]. Some researchers realized that taking historical traffic in spectrum sensing bands into account will lead to better spectrum sensing techniques [5], [31], [4], [37]. Such approach was confirmed to lead to better spectrum sensing results.When a wideband (large number of channels) of spectrum needs to be sensed for utilization by secondary users, available sensing time needs to be optimally utilized [14], [15], [16], [17]. Using this intuitive idea, a precise research problem was formulated and solved in [9] (keeping practical implementation constrains in mind).

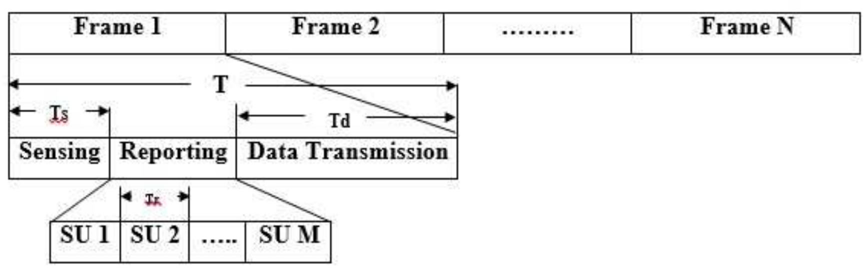

Fig. 2. Sensing and Data Transmission Time

On closer examination it is realized that from spectrum sensing implementation view point, it is natural to utilize only two spectrum sensing times: High/Low sensing times. This variation led us to the following problem formulation and its solution. We explain the research contribution, below in various stages. Fig.1 represents the spectrum in use and spectrum holes being utilized by secondary user. Fig.2 represents the total time allocation (T) for a frame i.e. for sensing(Ts), reporting(Tr) and data transmission(Td), where $T = Ts + Tr + Td$ [25], [26], [27], [33], [35], [36], [39].

- **Modeling Assumptions**

Case I: *Total number of bands are specified.*

- Let the total available sensing time for sensing a wide band be ”L”.

• Let the low sensing time be ’a’ and the high sensing time be ’b’.

• Let the number of bands in which the sensing time is ’a’ be ’x’ and the number of bands in which the sensing time is ’b’ be ’y’.

• Let the total number of bands be ’M’ i.e. $x + y = M$. Thus, we require that

$$ax + by = L \tag{1}$$

with the constraint $x + y = M$.

Thus x,y are unknowns and a, b are known sensing times.

We have a simple linear Diophantine equation for which exact integer solution for {x, y} exists under suitable condition. When such a solution exists, it is clear that the solution to this problem is trivial i.e.

$$ax + b(M - x) = L$$

$$(a - b)x = L - bM$$

If x = $\frac{L-bM}{a-b}$ is an integer, exact unique solution for {x, y} exists. If not, the approximate solution is obtained using

$$x = \lfloor \frac{L - bM}{a - b} \rfloor,$$

where $\lfloor . \rfloor$ is lower ceiling function.

Case II: *Suppose the number of bands is not specified*

• The above two variable linear Diophantine equation is well studied [3] and if a single solution $(x_0, y_0)(ax_0 + by_0 = L)$ exists, there are infinitely many solutions. But, only finitely many solutions are practically meaningful and all other solutions can be eliminated. Suppose the Maximum number of allowed bands is $'M_0'$. All solutions $\{x^{'}, y^{'}\}$ for which $x^{'} + y^{'} \leq M_0$ are potential candidates for choice of number of bands in which sensing time is low, high respectively.

• Specifically, negative solutions, where $'x_0'$ and/or $'y_0'$ are negative can be eliminated. Further, if $x + y > M_0$ (where M_0 is the upper bound on number of bands), then all such solutions can be eliminated(since they are not practically meaningful).

We now provide a stochastic optimization based approach to arrive at a unique solution for number of bands in which sensing time is low or high (i.e. determination of ”optimal” number of bands with low or high sensing times $\tilde{x}, \tilde{y}$).

The optimization criteria can be one of the following

- Minimization of average number of bands (or)
- Maximization of average number of bands.

To arrive at average number of bands (utilized for sensing), we need the following concept.

• Traffic Probability Mass Function (PMF): Suppose there are 'N' bands with the historical traffic being available (i.e. number of packets i.e. $n_i^{\prime} s$ in bands). We arrive at a Probability Mass Function(PMF) in the following manner

$$p_i = \frac{n_i}{\sum_{j=1}^{M} (n_j)} \quad for \;\; 1 \leq i \leq N. \tag{2}$$

i.e.p_i's are traffic probabilities in various bands Let Z be the random variable taking integer values corresponding to number of bands in which sensing time is low/high with some probabilities[22],[23],[24]. It is clear that corresponding to each valid solution (x', y') of equation (1), the total number of bands, $x' + y'$ assumes certain values less than M_o. Also associated with each solution, the average number of bands utilized for spectrum sensing can be determined i.e. E[Z]

$$E[Z] = \sum n_i p_i = x'P + y'Q \tag{3}$$

where P, Q are sum of the traffic probabilities in bands with sensing time low or high. It readily follows that the minimum, maximum values of E[Z] depend on the relative values of {P, x, y} (since Q = 1-P). Since the traffic probability mass function can be arbitrary, optimal(in the above sense) solution to the above stochastic optimization problem can only be determined numerically. The numerical results are presented in Section IV.

Note: If $P < Q$ (based on traffic PMF), then $x > y$ is the solution which minimizes E[Z]. Suppose $(\hat{x}, \hat{y})$ is a solution of (1). To calculate the values of E[Z], Var[Z](in the tables provided below in section IV), p_i's are chosen to be sum of smallest $\hat{x}$ probabilities in the traffic PMF or the other way. Similarly Q_i's $(Q_i = 1 - P_i)$ are chosen.

Note: The determination of solutions (x, y) for $ax + by = L$ and the determination of traffic PMF are decoupled.

Note: The time optimal spectrum sensing problem formulated above has the following dual version.

III. DUAL SENSING PROBLEM

The number of bands with low or high sensing times i.e. x or y are known (in equation (1)). But the low or high sensing times i.e. a or b are unknown(The total integer valued low and high sensing times are not specified but are bounded by some number). This dual problem can be solved in the same manner as above problem. Details are avoided for brevity. The dual problem deals with Time Optimal Spectrum Sensing.

In summary, the solutions obtained from (1) (when there are more than one integer solution) are suitably utilized to determine x or y (a or b in the dual problem). Based on the traffic PMF, the bands in which sensing time is low or high are determined, to minimize/maximize E[Z] (and Var[Z]).

Lemma1: Let ”P” be the success probability of a Bernoulli Random Variable and let h(P) be its variance as a function of ’P’. h(P) constitutes a logistic map i.e. $h(P) = (x - y)^2 P(1 - P)$

Proof: We now express E[Z], Var[Z] in terms of {x, y, P}

$$E[Z] = xP + yQ = xP + y(1 - P) = y + (x - y)P$$

$$Var[z] = (x^2 P + y^2 Q) - (xP + yQ)^2$$

$$= x^2 P + y^2 (1 - P) - (xP + y(1 - P))^2$$

$$= x^2 P + y^2 - y^2 P - (xP - yP + y)^2$$

$$= x^2 P + y^2 - y^2 P - [(x - y)P + y]^2$$

$$= x^2 P + y^2 - y^2 P - [(x - y)^2 P^2 + y^2 + 2(x - y)yP]$$

$$= x^2 P - y^2 P - (x - y)^2 P^2 - 2(x - y)yP$$

$$= P[x^2 - y^2 - 2(x - y)y] - (x - y)^2 P^2$$

$$= P[x^2 - y^2 - 2xy + 2y^2] - (x - y)^2 P^2$$

$$= P[(x - y)^2] - (x - y)^2 P^2 = (x - y)^2 P(1 - P)$$

Q.E.D.

Variance as a function of 'P' is a logistic map ie.

$$Var_Z(P) = (x-y)^2 P(1-P) = h(P) \tag{4}$$

It is well known that h(P) is maximized for P =1/2. The maximum value of h(P) is $\frac{(x-y)^2}{4}$.

• Given P, var[Z] depends on differences of $x\prime$, $y\prime$ only.

• For a given value of P the solution ($x\prime$, $y\prime$) for which $x\prime$ - $y\prime$ is smallest will minimize variance of Z.

From the above expressions for E[Z], Var[Z], the following bounds are true.

$E[Z] \geq y$, number of bands in which sensing time is high(since $x > y$).

$$Var[Z] \geq (x_0 - y_0)^2 P(1-P) \tag{5}$$

$$Var[Z] \leq \frac{(x_0 - y_0)^2}{4},$$

(follows from well known logistic map result), where$(x_0, y_0)$are such that $(x_0 - y_0)$ is the smallest possible value (from among all solutions).

•**Connections to Source Coding:** The optimization problem solved in this paper has connections to Source Coding [8]. The source generates two alphabet symbols with probabilities (P, 1-P) [18], [19], [20], [21] The sequence of alphabet symbols generated constitute an Independent, Identically Distributed (I.I.D) random process. To minimize the average codeword length, we allocate more codeword symbols to the alphabet with lower probability (P) and higher codeword symbols to the alphabet with smaller probability. Based on the source-coding theorem (noiseless channel coding theorem), the average codeword length, $\bar{n}$ is bounded below by the Shannon entropy, H(Z) of the source i.e. [8].

$$\bar{n} \geq H(Z)$$

Huffman coding meets the lower bound.

IV. NUMERICAL RESULTS WITH EXAMPLES

Example1: If a = 20, b = 90,equation of interest is 20x + 90y = 200. The total sensing time = 200 msec. Low sensing time, a = 20 msec and High sensing time, b = 90 msec.

$$20x + 90y = 200$$

$$GCD(20, 90) = 10. \ (200 \ is \ divisible \ by \ 10.)$$

$$x = 1 + t(90/10)$$

$$y = 2 - t(20/10)$$

for t = ... -2, -1, 0, 1, 2, ... there are multiple solutions but there is only one interesting solution with t = 0. The one interesting solution i.e. $x = 1, y = 2$. Hence such a solution is unique.

Example2: Total sensing time is 360 msec. Low sensing time, a = 18 msec and High sensing time, b = 72 msec.
Equation:

$$ax + by = L$$

$$18x + 72y = 360$$

$$GCD(18, 72) = 18.(360 \ is \ divisible \ by \ 18.)$$

$$x + 4y = 20$$

one solution can be x = 0 and y = 0

$$x = 0 + t(72/18)$$

$$y = 5 - t(18/18)$$

Only for $t = 1, 2, 3, 4$ there are values with $\{x, y\} > 0$, so there are multiple solutions. Four solutions are (4,4), (8,3), (12,2), (16,1).
To build intuition into the achievable values of E[Z], Var[Z]; we now consider an example.

Example3: In the solutions in example-2, (8,3) =(x_0, y_0):

$$Var[Z] = 25p(1 - P) \leq 25/4, E[Z] = 3 + 5P \leq 8$$

For other solutions, we calculate E[Z], Var[Z].

$$(12, 2) => Var[Z] = 100P(1 - P)$$

$$E[Z] = 2 + 10P$$

$$(16, 1) => Var[Z] = 225P(1 - P)$$

$$E[Z] = 1 + 15P$$

e.g. If P = 1/15;

$$E[Z] = 3 + 5P = 3.33.., Var[Z] = 1.57$$

$$E[Z] = 2 + 10P = 2 + 0.66 = 2.66.., Var[Z] = 4.9$$

$$E[Z] = 1 + 15P = 1 + 1 = 2, Var[Z] = 14.$$

TABLE I

TRAFFIC PROBABILITY MASS FUNCTIONS FOR VARIOUS VALUES OF TRAFFIC

No.	Probability Mass Function																
Packets	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	p_{12}	p_{13}	p_{14}	p_{15}	p_{16}	p_{17}
2000	.025	.05	.062	.65	.07	.075	.087	0.1	.12	.15	.175	-	-	-	-	-	-
2000	.012	.025	.03	.32	.037	.05	.05	.062	.075	.1	.1125	.125	.1375	.15	-	-	-
2000	.012	.025	.025	.37	.37	.37	.05	.05	.62	.062	.075	.075	.075	.0875	.0875	.1	.1
3000	.03	.041	.041	.05	.05	.058	.091	.1	.133	.133	.266	-	-	-	-	-	-
3000	.008	.01	.015	.21	.025	.066	.07	.083	.091	.1	.108	.116	.133	.15	-	-	-
3000	.008	.01	.015	.02	.025	.033	.041	.05	.058	.058	.066	.716	.091	.1	.108	.166	.12
5000	.05	0.05	.06	.07	.08	.09	.1	.11	.12	.13	.14	-	-	-	-	-	-
5000	.02	.007	.04	.05	.05	.05	.06	.07	.08	.09	.1	.11	.12	.13	-	-	-
5000	.01	.015	.02	.03	.04	.045	.05	.05	.05	.06	.06	.07	.08	.09	.1	.11	.12

TABLE II

MEAN AND VARIANCE VALUES OF NUMBER OF BANDS

No.Packets	P_1	Q_1	x_1	y_1	$E_1[Z]$	$Var_1[Z]$
2000	.55	.45	8	3	5.75	6.1875
2000	.7125	.2875	12	2	9.125	20.48
2000	.9	.1	16	1	14.5	20.25
3000	.468	.532	8	3	5.34	6.22
3000	.717	.283	12	2	9.17	20.29
3000	.651	.349	16	1	10.765	51.12
5000	.61	0.39	8	3	6.05	5.94
5000	.75	.25	12	2	9.5	18.75
5000	.88	.12	16	1	14.2	23.76

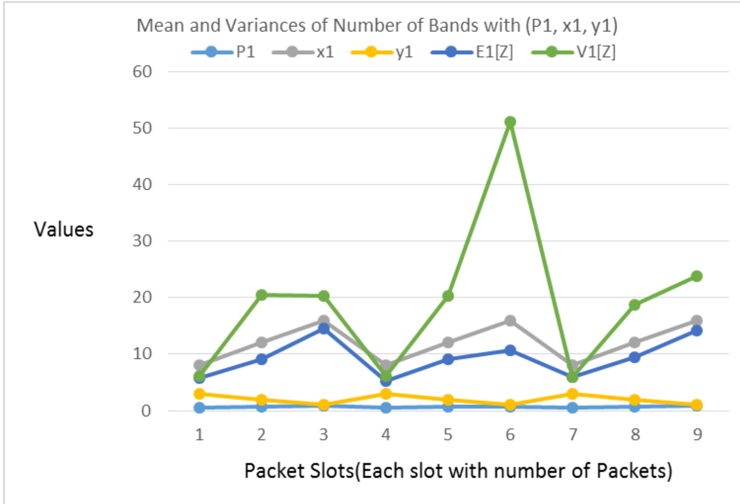

Fig. 3. Performance Measures of number of bands for sensing

TABLE III

MEAN AND VARIANCE VALUES OF NUMBER OF BANDS

No.Packets	P_2	Q_2	x_2	y_2	$E_2[Z]$	$Var_2[Z]$
2000	.55	.45	3	8	5.25	6.18
2000	.7125	.2875	2	12	4.875	20.48
2000	.9	.1	1	16	2.5	20.25
3000	.468	.532	3	8	5.66	6.22
3000	.717	.283	2	12	4.83	20.29
3000	.651	.349	1	16	6.235	51.12
5000	.61	0.39	3	8	4.95	5.94
5000	.75	.25	2	12	4.5	18.75
5000	.88	.12	1	16	2.8	23.76

We know that the solution which minimizes Var[Z] is the one for which $|(x_0 - y_0)|$ is minimum i.e.(x_0, y_0) (since $(x_0 > y_0)$). Let $(x_0 - y_0) = J$ and let the total number of sensing bands be $(x_0 + y_0) = K(K > J)$.

Hence

$$x_0 = \frac{J+K}{2} \quad and \quad y_0 = \frac{K-J}{2}.$$

For such a solution, we have that

$$Var[Z] = (x_0 - y_0)^2 P(1-P) = J^2 P(1-P))$$

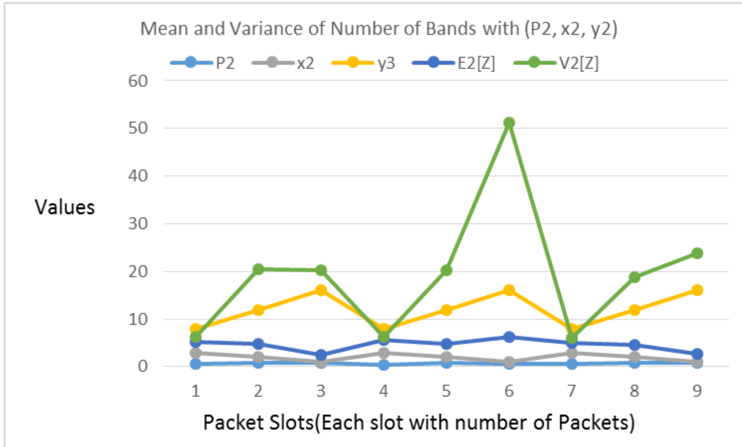

Fig. 4. Performance Measures of number of bands for sensing

TABLE IV

MEAN AND VARIANCE VALUES OF NUMBER OF BANDS

No.Packets	P_3	Q_3	x_3	y_3	$E_3[Z]$	$Var_3[Z]$
2000	.1375	.862	8	3	3.6875	2.96
2000	.037	.962	12	2	2.37	3.6
2000	.125	.875	16	1	2.875	24.6
3000	.1162	.8838	8	3	3.58	2.56
3000	.0183	.9817	12	2	2.18	1.79
3000	.0083	.991	16	1	1.124	1.85
5000	.16	0.84	8	3	3.8	3.36
5000	.095	.905	12	2	2.95	8.59
5000	.01	.99	16	1	1.15	2.22

$$E[Z] = \frac{K-J}{2} + JP$$

$$E[Z] = \frac{K}{2} + \frac{J(2P-1)}{2} = \frac{K}{2} + \frac{J(2P-1)}{2}. \tag{6}$$

Definition: We define a value of P, ”Pareto-optimal” solution if E[Z] = Var[Z] for that value of ’P’.

Lemma2: Pareto-optimal solution is (i.e. E[Z] = Var[Z]) not achievable for any probability ’P’ and integer values of x_0, y_0.

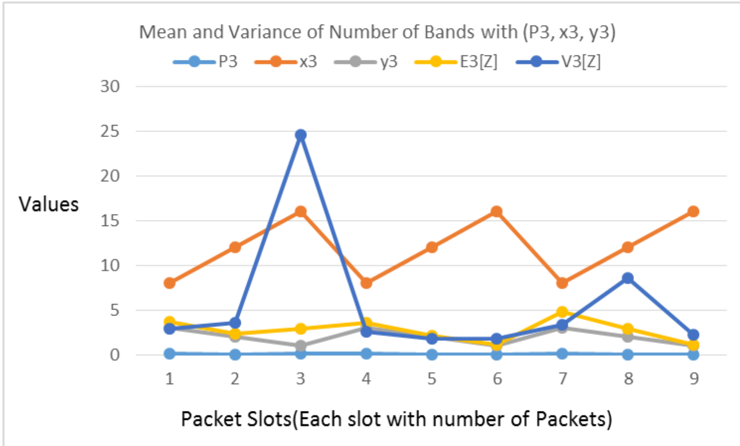

Fig. 5. Performance Measures of number of bands for sensing

TABLE V

MEAN AND VARIANCE VALUES OF NUMBER OF BANDS

No.Packets	P_4	Q_4	x_4	y_4	$E_4[Z]$	$Var_4[Z]$
2000	.1375	.862	3	8	7.31	2.96
2000	.037	.962	2	12	11.62	3.6
2000	.125	.875	1	16	14.12	24.6
3000	.1162	.8838	3	8	7.41	2.56
3000	.0183	.9817	2	12	11.81	1.79
3000	.0083	.991	1	16	15.875	1.85
5000	.16	0.84	3	8	7.2	3.36
5000	.095	.905	2	12	11.05	8.59
5000	.01	..99	1	16	15.85	2.22

Proof: Pareto optimal values of P can easily be computed in terms of {K, J}(as in the case of x,y)

$$J^2P(1-P) = \frac{K}{2} + \frac{J(2P-1)}{2}$$

$$2J^2P(1-P) = K + J(2P-1)$$

$$2J^2P(1-P) - K - J(2P-1) = 0$$

$$2J^2P(P-1) + K + J(2P-1) = 0$$

$$P^2(2J^2) + P(2J-2J^2) + (K-J) = 0$$

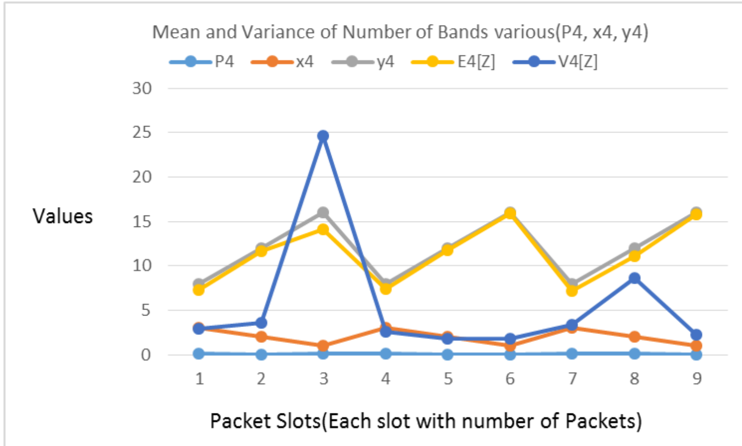

Fig. 6. Performance Measures of number of bands for sensing

The discriminant of the above quadratic polynomial is

$$= (2J(1-J))^2 - 4(2J^2)(K-J)$$

$$= 4J^2(1-J)^2 + 8J^3 - 8J^2K$$

$$= 4J^2(1+J^2-2J) + 8J^3 - 8J^2K$$

$$= 4J^2 + 4J^4 - 8J^3 + 8J^3 - 8J^2K$$

$$= 4J^2 + 4J^4 - 8J^2K = 4J^2(1+J^2) - 8J^2K$$

$$= 4J^2(1+J^2) - 4J^2(2K)$$

i.e. Pareto-optimal solution is achievable only if

$$1 + J^2 \geq 2K$$

i.e.

$$1 + (x_0 - y_0)^2 \geq 2(x_0 + y_0)$$

$$1 + x_0^2 + y_0^2 - 2(x_0 + y_0 + x_0 y_0) \geq 0 \tag{7}$$

We readily have that by arithmetic - geometric mean inequality

$$\frac{x_0 + y_0}{2} \geq \sqrt{x_0 y_0}$$

$$(x_0 + y_0)^2 \geq 4x_0 y_0$$

$$x_0^2 + y_0^2 + 2x_0y_0 \geq 4x_0y_0$$

$$x_0^2 + y_0^2 \geq 2x_0y_0$$

$$1 + x_0^2 + y_0^2 \geq 1 + 2x_0y_0$$

$$1 + x_0^2 + y_0^2 - 2(x_0 + y_0 + x_0y_0) \geq 1 + 2x_0y_0 - 2(x_0 + y_0 + x_0y_0)$$

$$\geq 1 - 2(x_0 + y_0) \geq 0$$

i.e. can happen if

$$(x_0 + y_0) \leq 1/2$$

Q.E.D

It is impossible since $''x_0 + y_0''$ is an integer larger than '1'.

Lemma3: Given a value of P, if (x_0, y_0) minimizes Var[Z], then it will maximize E[Z]. Conversely, if E[Z] is minimized, then Var[Z]is maximized.

Proof: It readily follows that Var[Z] is minimized when $(x_0 - y_0) = J$ assumes minimum possible value. But

$$E[Z] = y_0 + (x_0 - y_0)P. \tag{8}$$

$$E[Z] = \frac{K - J}{2} + JP \quad where \quad K > J, \tag{9}$$

$$Var[Z] = J^2P(1 - P) = JP(J(1 - P)) \tag{10}$$

If J is minimized and K assumes maximum possible value, E[Z] is maximized. Also, the converse holds true.

Q.E.D.

Now for the sake of completeness, we present numerical results. For that purpose we specify the traffic probability mass function for various total number of packets and total number of bands.

Note: In view of the above derivation E[Z] = Var[Z] is not achievable.

Note: Suppose the number of packets in bands with low sensing time is known. Then, the value of P from traffic PMF is a given fixed value. This corresponds to a simple thresholding scheme to determine number of bands with high/low sensing time.

Note: For the solution (x_0, y_0) of Table I, maximum possible value of $x_0 + y_0$ is 17.i.e. Table I specify the traffic PMF chosen. Tables II, III, IV, and V calculate E[Z], Var[Z] for various traffic PMF's i.e.values of P,Q. Fig.3, Fig.4, Fig.5 and Fig.6 represents the mean and variances for various traffic probabilities i.e. P, different x and y values.

V. CONCLUSION

In this research paper, an optimal spectrum sensing problem called Bang Bang sensing is formulated and solved. It readily follows that a dual problem can be solved by similar approach. Some formal lemmas on number of spectrum bands are proved. Some numerical results are provided.

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