Algorithms for Designing Communication Networks Using Greedy Heuristics of Various Types

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Abstract: We consider two frequently arising problems in the modeling of a communication network related to the construction of a graph of a communication network that satisfies certain conditions. An algorithm is proposed to which the solution of both problems related to the class of greedy algorithms can be reduced. The question of the uniqueness of the solution of the tasks is investigated. A positive result of solving the problem is obtained and sufficient conditions for uniqueness are identified. The research and development of the corresponding software are of practical importance in the design of real communication networks.

1. Introduction

Consider the task of building a communication network based on the disparate fragments of a communication network. Such a task often arises during network modernization and is accompanied by a set of analytical studies to identify and achieve the required technical characteristics by the communication network. In this case, we will consider the construction of a connected graph of a communication network on the basis of existing disconnected subgraphs of the network with the requirement to minimize construction costs, which corresponds to the minimum value of the sum of the lengths of the edges of the simulated graph.

In fact, the situation under consideration is the task of constructing a spanning tree of minimum weight [1]. We will solve this problem using the heuristic greedy algorithm. Two varieties of this problem will be considered (hereinafter we will call them Task 1 and Task 2), which is of rather high practical significance in the field of the theory of communication networks [2]. When constructing a mathematical model of a communication network in terms of solving tasks 1,2, it is proposed to use one general algorithmic module based on a greedy algorithm. In addition, the following publications will provide rigorous proof of the uniqueness of a solution under certain conditions (see below for sufficient conditions for uniqueness). Note that in the general case, according to Kirchhoff's theorem [1, p. 57], there can exist more than one spanning tree in a connected graph. To find the skeleton of minimum weight, Kruskal and Prima algorithms can be used [1, p. 60]. In our case, the existing graph of the communication network needs to be completed to such a graph for which all newly constructed edges would be elements of a spanning tree, which for the constructed graph is determined using the above algorithms. The following algorithms can be considered as adaptations of the Kraskal algorithm to the task of constructing an optimal communication network with the introduction of novelty of uniqueness conditions for the optimal solution. Adaptation is subject to the Kruskal algorithm being applied to some complete graph constructed on the basis of the existing graph of the communication network.

2. Task 1. Construction of a communication network based on a given source communication network that satisfies the condition of connectivity and the minimum total length of the completed communication lines

The problem statement looks like this. Let be $G =< V, E >$ source graph of the communication network. We have an optimization problem of the following type:

$$\sum_{i=1}^{N_{G'}} \rho(r_i) \to \min \tag{1}$$

$$\forall d_i \in \widehat{D}\ f(d_i) > 0,\ i \in \left[1, N^{\widehat{D}}\right] \tag{2}$$

Here r_i is building a line of communication, $\rho(r_i)$ is the length of the communication line r_i, $\rho(r_i) \in R$ it is a function of the geographical distance between points with specified geographical coordinates determined by the incident vertices of the communication line r_i, $\widehat{D} = \{d_i\}_{i=1}^{N_{\widehat{D}}}$ is a set of all possible information communication directions [3], $N_{\widehat{D}} \in N$ is a number of all possible communication directions ($N_{\hat{D}} = C_{N_{I_Z}}^2$, $N_{\hat{D}} = \dfrac{(N_{I_Z})!}{2!(N_{I_Z}-2)!}$), $d_i = (v_1^{(i)}, v_2^{(i)})$ is the information direction of communication, where $v_1^{(i)} \in V$ (identifier) and $v_2^{(i)} \in V$ (identifier) form an offsetting pair, $f(d_i)$ is reliability of the information direction of communication d_i [3-5], $d_i \in \widehat{D}$, $f(d_i) \in [0,1]$, $f(d_i) \in R$. In other words, you need to build a connected graph for a given source graph with the minimum total length of the completed edges. The optimal solution of the problem is found using a greedy algorithm. Among pairs of graph vertex (v_1, v_2), где $v_1 \in V$, $v_2 \notin V$, not yet included in the set of edges, such is chosen, the first one, vertex v_1 and v_2 in an already constructed graph belong to different connected components (i.e. $f(d) = 0$, where $d \in \hat{D}$, $d = (v_1, v_2)$), and, the second one, among all such pairs, one is selected for which the value $\rho(v_1, v_2)$ is minimal.

3 Algorithm for solving Task 1

Step 1. For all edges of the original graph, put the weight coefficient equal to zero.

Step 2. We set the set of edges to be $M := \otimes$

Step 3. We set the auxiliary set of vertices to $V' = \{v_1\}$, $v_1 \in V$.

Step 4. We set $V := V - V'$, i.e. exclude a vertex $v_1 \in V$ from the set V.

Step 5. If $V = \otimes$, then end. Else go to step 6.

Step 6. Choosing a pair of graph vertices (v_1, v_2) satisfying the condition:

1) $v_1 \in V'$ and $v_2 \in V$
2) $(v_1, v_2) \in E$
3) $v_1 \neq v_2$
4) $(v_1, v_2) \notin M$

If there exists a pair of graph vertices, then we assume

1) $V := V - \{v_2\}$,
2) $V' := V' \cup \{v_2\}$,
3) $M := M \cup \{(v_1, v_2)\}$
4) $E := E - \{(v_1, v_2)\}$

Step 7. Choosing a pair of graph vertices (v_1, v_2) satisfying the condition:

1) $v_1 \in V'$ and $v_2 \in V$
2) $v_1 \neq v_2$
3) $(v_1, v_2) \notin M$
4) $\rho(v_1, v_2)$ is minimum among all $\rho(v_1, v_2)$ satisfying the condition $(v_1, v_2) \in E$

If there exists a pair of graph vertices, then we assume

1) $M := M \cup \{(v_1, v_2)\}$,
2) $E := E - \{(v_1, v_2)\}$
3) $V := V - \{v_2\}$,
4) $V' := V' \cup \{v_2\}$.

Step 8. If $V = \otimes$, then end, M is the desired set of edges of the graph, which provides connectivity and the minimum total length. Else go to step 6.

Statement 1. The algorithmic complexity of the algorithm for solving task 1 is $O(|V|^3 \times |E|)$.

Proof. To join the next vertex, you need to look through the list of all available edges of the original graph, therefore, the complexity will be proportional to $|E|$ (in fact, it is possible to optimize the enumeration of edges by viewing only those that do not yet connect the vertices in the graph already constructed). Consider the procedure for enumerating the vertices of a graph to attach another vertex. Let be n is a number of vertices in a graph and i is a number of vertices already attached. To join the next $(i+1)$-th vertex the analysis of pairwise vertices from sets of cardinalities i and $n - i$. From here the number of pairs analyzed will be $\sum_{i=1}^{n}[i(n-i)]$. Convert this expression. We get

$n\sum_{i=1}^{n} i - \sum_{i=1}^{n} i^2 = n^2(n+1)/2 - n(n+1)(2n+1)/6 = n(n+1)(n-1)/6$.

Therefore, we have the cubic complexity of the power of the set of vertices. Given the proportionality of the algorithmic complexity of the cardinality of the set of edges of the graph $|E|$, we obtain the asymptotic $O(|V|^3 \times |E|)$.

The following statement is proved, which gives the sufficiency of uniqueness of the optimal solution to Task 1.

Statement 2. If the distances between non-incident vertices of the graph are different, then this algorithm leads to the only optimal solution to Task 1.

Note that the difference in pairwise distances between the vertices of the graph is easily achieved due to the high resolution of the real number (for example, for the floating-point number format in the IEEE 754 standard, the possible range of numbers is from $4{,}94 \cdot 10^{-324}$ to $1{,}79 \cdot 10^{308}$), which represents the distance between the vertices and takes place on real-time communication networks.

Consider another problem that often arises when designing / modeling a communication network. Namely, this is the task of optimal binding of consumer nodes to nodes of communication providers. Optimality here will also be considered with respect to the minimum total length of the ribs being completed. Thus, a graph should be composed of many subgraphs (not necessarily interconnected), such that each subgraph must contain exactly one element of the set $I_M = \{s_i^M\}_{i=1}^{N_{I_M}}$, and the total length of the edges should be minimal.

4. Task 2. Building bindings of nodes-consumers of the communication network to nodes-providers, satisfying the condition of the minimum total length of the completed communication lines

The statement of the problem is as follows. Let be $G = \langle V, E \rangle$ source graph of the communication network, moreover, $V = I_M \cup I_Z$, where $I_M = \{s_i^M\}_{i=1}^{N_{I_M}}$ - is the set of nodes of communication providers, $N_{I_M} \in N$; s_i^M - communication provider node; $I_Z = \{s_i^Z\}_{i=1}^{N_{I_Z}}$ - many communication consumer nodes, $N_{I_Z} \in N$, s_i^Z - many communication consumer nodes, $N_{I_Z} \in N$. Optimization task:

$$\sum_{i=1}^{N_{G'}} \rho(r_i) \to \min \tag{3}$$

$$G' = \bigcup_{i=1}^{N_{I_M}} \langle V'(i), E'(i) \rangle \quad \text{, где} \tag{4}$$

$V'(i)$ contains exactly one element from the set $I_M = \{s_i^M\}_{i=1}^{N_{I_M}}$, and no element $s_i^M \in I_M$ can be included in different sets $V'(i)$ и $V'(j)$ ($i \neq j$). That is, there is a bijective correspondence of elements of sets $I_M = \{s_i^M\}_{i=1}^{N_{I_M}}$ и $\{V'(i)\}_{i=1}^{N_{I_M}}$.

Moreover $E'(i)$ it contains at least one edge incident

$s_i^M \in I_M \quad (i = 1..N_{I_M})$. (5)

This task can also be successfully solved using the greedy algorithm. For this, it is necessary to carry out algorithmic adaptation of the solution of problem 2 to the solution of task 1. As a result, the algorithm will be as follows.

5 Algorithm for solving Task 2

1. For all edges of the graph, put the weight coefficient $\rho(v', v'')$ equal to zero.
2. We assume that there are many edges to be completed $M := \otimes$
3. We assume an auxiliary set of vertices $V' = I_M$, that is, a set of vertices corresponding to provider nodes.
4. We assume $V = I_Z$, that there are many peaks corresponding to consumer nodes.
5. Next, we use the algorithm for solving task 1, starting from step 5.

The implementation of this algorithm will ensure that the requirements (3-5) are met. Algorithmic complexity will also be $O(|V|^3 \times |E|)$ (see Proposition 1).

Concerning task 2, it can also be proved that the greedy algorithm given will give the only optimal solution if all pairwise distances between the vertices are different.

6. Conclusion

Thus, based on two facts - low algorithmic complexity, acceptable for modern computing resources, as well as the proven ability to provide the only optimal solution in this case, the probability of which can be reduced to unity, greedy algorithms are an effective tool for building and / or upgrading networks communication. The proposed algorithms were implemented in the construction of real large-scale communication networks, and can accordingly be used in solving problems of constructing a complete communication network with the condition of minimizing the built-up communication lines, and in solving problems of optimal connection of communication network consumer nodes to provider nodes.

References:

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