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## Abstract

The occurrence of vibrations during a roller coaster ride can significantly impact the overall experience and contribute to structural fatigue. Despite its relevance, understanding the root causes or accurately predicting these vibrations remains an unresolved challenge, introducing an element of unpredictability into the design process.

Some investigations have elucidated the role of rail irregularities in the vibration response of roller coaster systems [1, 6]. Advancements in the rail manufacturing and bending processes have proven to enhance the ride smoothness due to the reduction of deviations with respect to the designed trajectory. Moreover, the use of shock absorbers further dampens vibrations and impacts due to contact clearances. However, it is essential to recognize that significant vibrations are still present in modern rides.

Expanding the scope of possible causes for these vibrations, in [3, 4, 5] the authors suggest that friction-induced self-excited vibrations may manifest in roller coaster systems in absence of irregularities, even though notable levels of vibration were observed in the proposed models when wheel-rail clearance was present, and in general lower levels with vanishing clearance. Here, the combined effect of the proposed contact model in the self-excited analysis and the rail irregularities is studied.

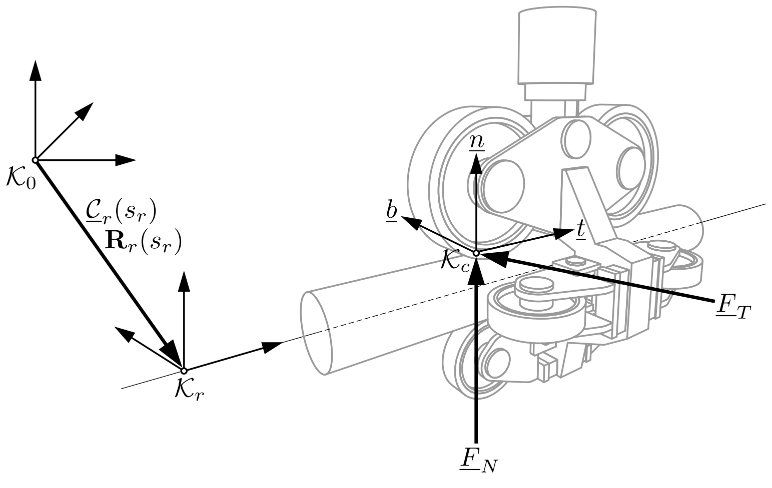


Figure 1: Schematic representation of the wheel-rail contact point and forces acting upon the rear running wheel.

Figure 1 illustrates a wheel bogie of a roller coaster car. The rail trajectory is represented by the curve $\underline{\mathscr{C}}_r(s_r)$ and the rotation matrix $\mathbf{R}_r(s_r)$, both parametrized with respect to the rail arc length $s_r$. $\mathscr{K}_r$ has one axis tangent to the rail centerline and one axis perpendicular to the opposite rail. The contact interface, represented by the coordinate system $\mathscr{K}_c$, is found by imposing a cylinder-cylinder contact constraint between the wheel and the rail surfaces. The normal $\underline{F}_N$ and tangential $\underline{F}_T$ contact forces are assumed of the form

$$\underline{F}_N = \begin{cases} \left(k\delta^p + c\delta^{p-1}\dot{\delta}\right)\underline{n} & \text{for } \delta > 0, \\ \underline{0} & \text{otherwise,} \end{cases} \tag{1}$$

$$\underline{F}_T = \begin{cases} \mu \left\|\underline{F}_N\right\| \tanh\left(\frac{\left\|\underline{F}'_T\right\|}{\mu \left\|\underline{F}_N\right\|}\right) \frac{\underline{F}'_T}{\left\|\underline{F}'_T\right\|} & \left\|\underline{F}_N\right\| > 0 \text{ and } \left\|\underline{F}'_T\right\| > 0 \\ \underline{0} & \text{otherwise,} \end{cases} \tag{2}$$

where $k$ is a stiffness constant, $c$ is a damping constant, $\delta$ is the rigid interpenetration depth between the wheel tread and the rail, $p$ is a parameter defining the contact stiffness depending on $\delta$, $\mu$ is the friction coefficient depending on the sliding speed between the contact point on the wheel tread and the rail surface and $\underline{F}'_T$ is the creepage-dependent tangential force without taking into account saturation effects, which in this case is given by Kalker's linear theory.

A deviation profile $\underline{\tilde{\delta}}(s_r) = [0, \tilde{\delta}_y(s_r), \tilde{\delta}_z(s_r)]^T$, expressed in $\mathscr{K}_r$, is added to the ideal contact point (without irregularities), generated by a stationary ergodic Gaussian process in space [2].

In Figure 2, the simulated vibration levels of the third car of a four-car roller coaster train are shown, in the presence of friction, rail irregularities, and contact clearance. The evolution of the frequencies associated with more prominent vibration levels is not random. Rather, it has a direct relationship with the linearized stiffness, that depends on $\delta$, that is, on the dynamic profile of the roller coaster. This phenomenon, attributed to the nonlinear nature of the contact, is also observed in real systems.

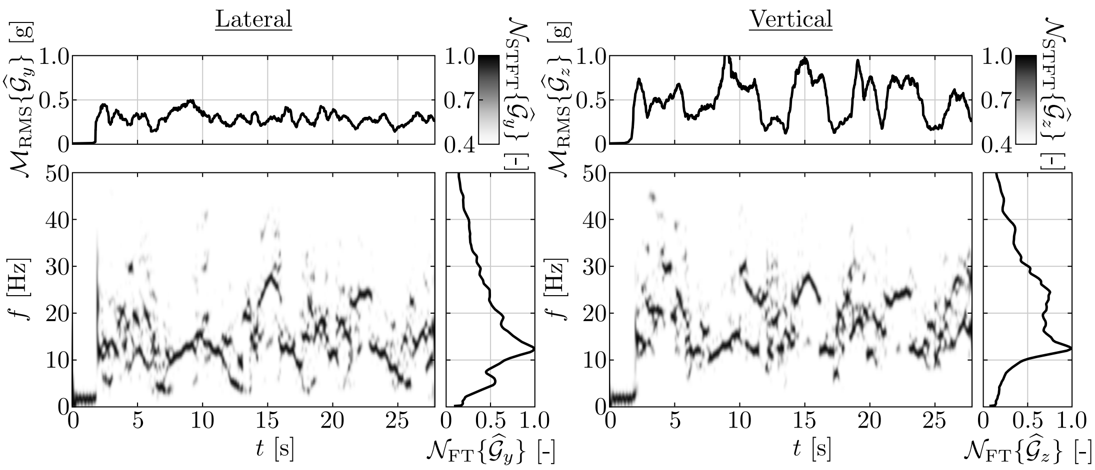


Figure 2: Spectrograms of the simulated vibration response of a roller coaster car.

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