# How to Estimate Web Application Concurrent Events Using Either Load Generator Data or Queueing Model Results

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# How to Estimate Web Application Concurrent Events Using Either Load Generator Data or Queueing Model Results

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*Some of the most sought-after web application load testing results are the concurrent GET/POST event quantities used to dimension internal system resources such as process thread pools and input queue buffers. The concurrency information desired is the proportion of time there are 0, 1, 2, etc. web events resident in the system under test. These proportions are not always available from target system measurements but, as will be shown, can be approximated with data produced by the load generator, or estimated using queueing models. The data analysis and queueing model tools that support the ideas presented were developed by this author and contained in two of his GitHub repositories, web-generator-toolkit2 and QueState, as free open source downloads.*

## 1.0 Introduction

One of the many questions load testing professional are asked is, "how many concurrent users will the new web application support?" This question normally refers to the number of user requests that can simultaneously be resident in the System Under Test (SUT) while maintaining acceptable service levels. Often, there are no direct measurements being performed in the SUT to draw a picture of concurrency, but load tool query statistics as well as queueing model state probabilities can be used to estimate concurrency levels. The load tool approach taken here is to combine query timestamps produced by the load generator with their associated round-trip response times to manufacture an estimate of user request concurrency. The queueing model state probabilities are typically useful as a concurrency estimator for applications that are in the early stages of design or development before measurements are possible.

The web-generator-toolkit2 (Toolkit2) data analysis software on GitHub is this author's approach to concurrency distribution approximation using load tool data. It provides the statistical count and proportion of time there are "N" user requests in the system being tested and it does this for specific query types as well as the total number of requests outstanding. The example used in this document to illustrate the ideas presented is the one contained in the Toolkit2 demo.

The queueing model state probabilities are produced with the QueState package on GitHub. This repository contains eight queueing models and is somewhat unique in that it lists state probabilities along with the usual performance statistics like average waiting time in system, i.e., mean response time. The state probabilities produced by these computer programs are a modeling estimator of the request concurrency distribution.

The discussion begins in Section 2 with a pictorial representation of concurrent events within the context of a load testing environment using the Toolkit2 demo as a basis. Section 3 expands upon this demo example with additional load test specifics used to step through the concurrency calculations performed by the analysis program. This section also summarizes the concurrency related results that program produces. This summary information leads to the Section 4 queueing model discussion where Toolkit2 demo traffic parameters are applied to two candidate models with a graphical comparison made between the model results and demo statistics. Section 5 wraps up the discussion with some conclusions and summary remarks.

### 2.0 Load Generator View of Concurrent Events

Figure 1 shows a load testing environment containing a traffic generator on the left and the System Under Test on the right with the two of them communicating over Ethernet. The load generator is configured with 200 virtual user threads operating in closed loops offering web traffic to the SUT containing 8 CPU cores. There are nine GET/POST web events initiated by the traffic generator currently being processed in the SUT.

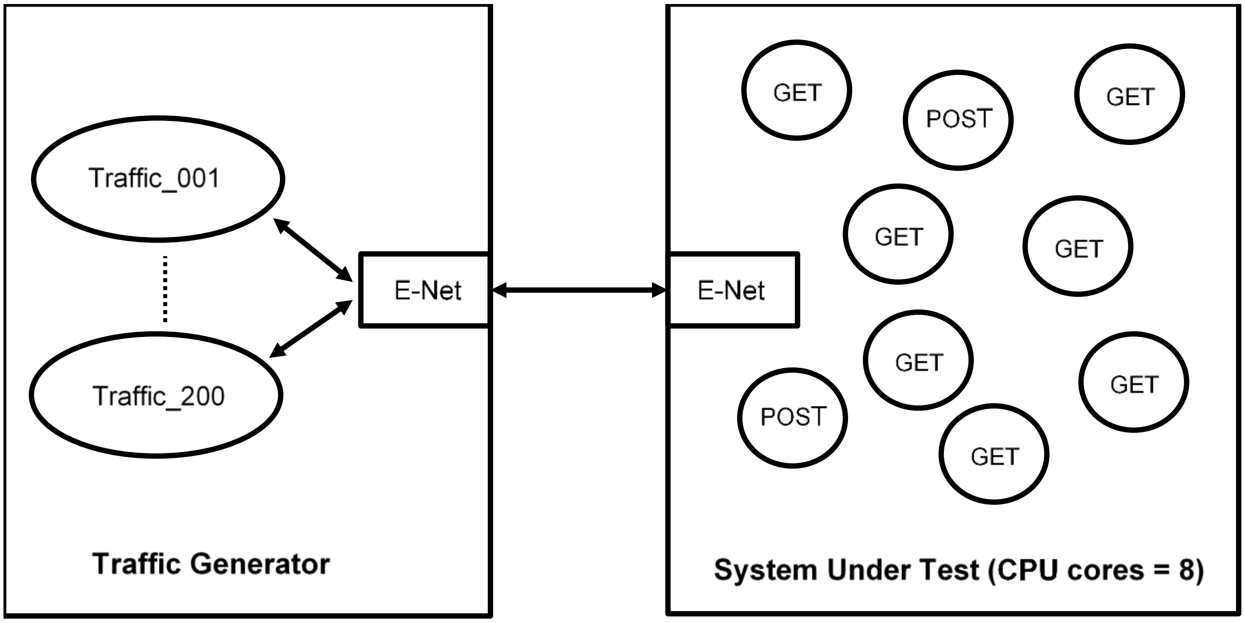
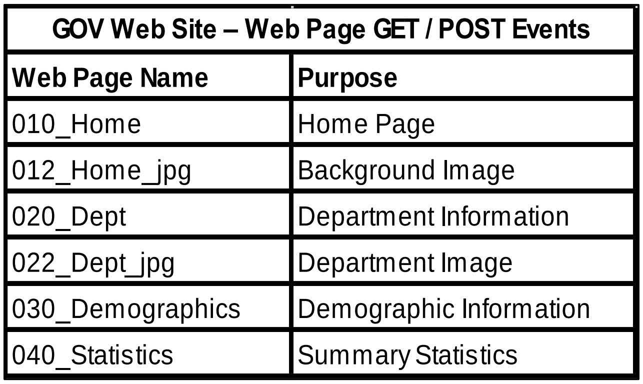


**Figure 1: Load Generator and System Under Test (SUT) Topology**

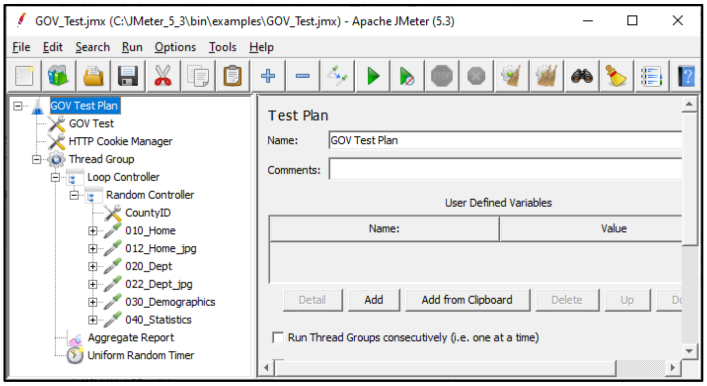
The concurrency information of interest is a list containing the fraction of the time the SUT is processing 0, 1, 2, ... 9, …, etc. web events. When this information is not available from the SUT, how can query/response information collected by the load generator be used to estimate the SUT’s concurrency distribution? To clearly describe the steps taken to produce SUT concurrency statistics from load generator data the topological setup of Figure 1 is extended into the load testing specifics of Figure 2.

### 3.0 Concurrency Calculation Setup

Figure 2 contains the Toolkit2 demo list of web events being tested on the left and the layout of the JMeter load testing script on the right. Six events are being queried to obtain state government statistics with a single group containing 200 virtual user threads. Threads randomly select an event from the list each cycle (think + response) and draw their think times from a uniform random timer.



| GOV Web Site – Web Page GET / POST Events | |
|---|---|
| **Web Page Name** | **Purpose** |
| 010_Home | Home Page |
| 012_Home_jpg | Background Image |
| 020_Dept | Department Information |
| 022_Dept_jpg | Department Image |
| 030_Demographics | Demographic Information |
| 040_Statistics | Summary Statistics |



GOV_Test.jmx (C:\JMeter_5_3\bin\examples\GOV_Test.jmx) - Apache JMeter (5.3)

```
GOV Test Plan
    GOV Test
    HTTP Cookie Manager
    Thread Group
        Loop Controller
            Random Controller
                CountyID
                010_Home
                012_Home_jpg
                020_Dept
                022_Dept_jpg
                030_Demographics
                040_Statistics
        Aggregate Report
        Uniform Random Timer
```

Test Plan

Name: GOV Test Plan

Comments:

User Defined Variables

| Name: | Value |
|---|---|

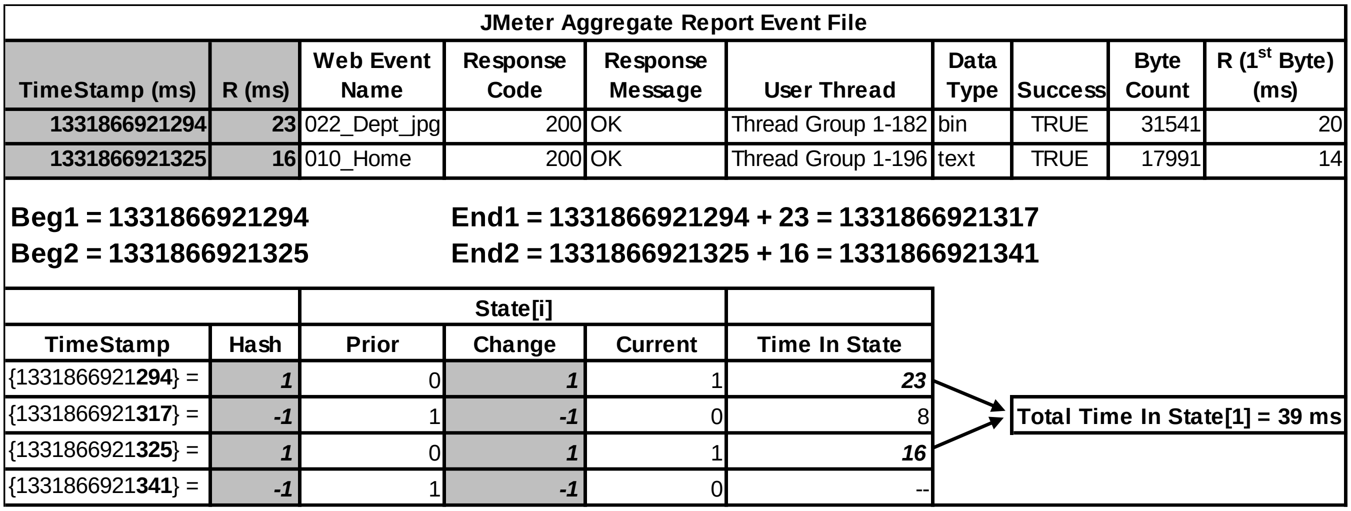
☐ Run Thread Groups consecutively (i.e. one at a time)

**Figure 2: Web Page Events and JMeter Load Testing Script**

The test run is 25-minutes in duration with the first 2 and last 3 minutes excluded from the analysis to minimize the impact of startup and shutdown time transients. The resulting JMeter Aggregate Report output file, 2000_AggRpt_120_1199.csv, contains 92,984 query/response records. The first two rows of that file are listed at the top of Figure 3 where the timestamp and response time fields are shown in gray. The TimeStamp (ms) column is the launch time of the event in Unix time expressed in milliseconds and the R (ms) is the elapsed milliseconds required for the response to be returned. The term “State” refers to the number of concurrent user queries estimated to be resident in the SUT. The remainder of this figure uses the two records shown to illustrate how concurrency calculations are performed within the entire file.

### 3.1 Concurrency Calculation Methodology

The first step implemented in the Toolkit2 software is to create End timestamps by adding the response time to the Beg timestamp. A previously unset Beg timestamp hash is assigned a value of 1 and an unset End timestamp hash is given a value of -1. Subsequent occurrences of a timestamp cause its hash value to be incremented by one if it is a Beg occurrence and decremented by one if it is an End occurrence. When this analysis is complete, each timestamp has a single hash number assigned to it that represents its state change contribution. The timestamp list is then processed in sorted timestamp order where Prior State + Change State = Current State. For simplicity Figure 3 only shows (+1) or (-1) for a given timestamp’s hash/change value but that number is often greater than +1 or less than -1 when the entire file is processed.



JMeter Aggregate Report Event File

| TimeStamp (ms) | R (ms) | Web Event Name | Response Code | Response Message | User Thread | Data Type | Success | Byte Count | R (1st Byte) (ms) |
|---|---|---|---|---|---|---|---|---|---|
| 1331866921294 | 23 | 022_Dept_jpg | 200 | OK | Thread Group 1-182 | bin | TRUE | 31541 | 20 |
| 1331866921325 | 16 | 010_Home | 200 | OK | Thread Group 1-196 | text | TRUE | 17991 | 14 |

**Beg1 = 1331866921294** **End1 = 1331866921294 + 23 = 1331866921317**
**Beg2 = 1331866921325** **End2 = 1331866921325 + 16 = 1331866921341**

| | | State[i] | | | |
|---|---|---|---|---|---|
| TimeStamp | Hash | Prior | Change | Current | Time In State |
| {1331866921**294**} = | *1* | 0 | *1* | 1 | ***23*** |
| {1331866921**317**} = | *-1* | 1 | *-1* | 0 | 8 |
| {1331866921**325**} = | *1* | 0 | *1* | 1 | ***16*** |
| {1331866921**341**} = | *-1* | 1 | *-1* | 0 | -- |

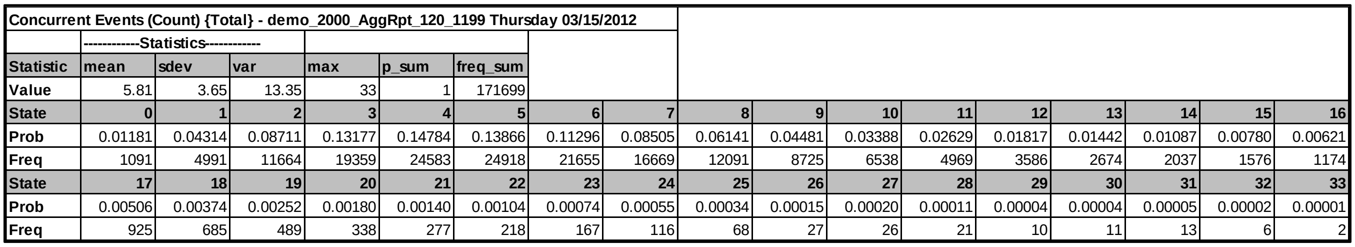
**Total Time In State[1] = 39 ms**

**Figure 3: Concurrent Query Calculations Using Load Generator data**

The time in a particular State is calculated by subtracting the current timestamp from the next one in ascending order. Those calculated values are summed for each State with the example in Figure 3 yielding a total time in State[1] = 39 milliseconds. This methodology produces an array of States[ ], i.e., [0], [1], [2], etc., with each element equal to its total milliseconds. Dividing these millisecond sums by the total milliseconds for the test yields the proportion of time in each State. This is the distribution of concurrent events in the SUT estimate using measurements taken by the load generator. Obviously, long network latencies across the E-Net connection between the load generator and the SUT will skew this estimate. See Appendix A for a concurrency function source listing.

### 3.2 Concurrency Calculation Results

Figure 4 is the resulting comma delimited output file saved as a spreadsheet for the Toolkit2 demo example where data for all six web events, 010_Home through 040_Statistics, are included in the calculations. It contains a statistical “Value” row, a state transition count by state, “Freq” set of numbers, and a proportion of time in each state, “Prob” list of empirical state probabilities. The “Value” row mean, sdev, and var are calculated with the “State” and “Prob” information using the formulas for grouped data, e.g., $mean = \sum State\ x\ Prob$ = 5.81 concurrent events. This row also lists the maximum number of concurrent queries in the SUT during the entire measurement period, max = 33, and the total number of state change timestamps processes, freq_sum = 171,699.



| Concurrent Events (Count) {Total} - demo_2000_AggRpt_120_1199 Thursday 03/15/2012 | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ------------Statistics------------ | | | | | | | | | | | | | | | | |
| **Statistic** | **mean** | **sdev** | **var** | **max** | **p_sum** | **freq_sum** | | | | | | | | | | | |
| **Value** | 5.81 | 3.65 | 13.35 | 33 | 1 | 171699 | | | | | | | | | | | |
| **State** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** |
| **Prob** | 0.01181 | 0.04314 | 0.08711 | 0.13177 | 0.14784 | 0.13866 | 0.11296 | 0.08505 | 0.06141 | 0.04481 | 0.03388 | 0.02629 | 0.01817 | 0.01442 | 0.01087 | 0.00780 | 0.00621 |
| **Freq** | 1091 | 4991 | 11664 | 19359 | 24583 | 24918 | 21655 | 16669 | 12091 | 8725 | 6538 | 4969 | 3586 | 2674 | 2037 | 1576 | 1174 |
| **State** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** | **29** | **30** | **31** | **32** | **33** |
| **Prob** | 0.00506 | 0.00374 | 0.00252 | 0.00180 | 0.00140 | 0.00104 | 0.00074 | 0.00055 | 0.00034 | 0.00015 | 0.00020 | 0.00011 | 0.00004 | 0.00004 | 0.00005 | 0.00002 | 0.00001 |
| **Freq** | 925 | 685 | 489 | 338 | 277 | 218 | 167 | 116 | 68 | 27 | 26 | 21 | 10 | 11 | 13 | 6 | 2 |

**Figure 4: Concurrent Events by Proportion of Time in State (Prob) and Frequency Count (Freq)**

The proportion of time across all states experienced (0 – 33) is the concurrency state probability distribution based on this set of measurements. It should be noted that these empirical state probabilities differ from the frequency count by state fraction of total values because they account for time in a state. For example, the frequency count, “Freq”, decimal fraction for state zero is Freq[0] = 1091 / 171699 = 0.00635, but its empirical state probability is Prob[0] = 0.01181. The GitHub location of the Toolkit2 demo output file that produces the Figure 4 results is provided in Appendix B.

Figure 5 contains summary information for three of the Toolkit2 demo statistical reports. This figure includes results for all six web events and is the “Total” row of the complete set of reports shown in Appendix B.

1. Concurrent Events
2. Response Times
3. Time Between Queries

| **Summary Statistics (ms) - demo_2000_AggRpt_120_1199 Thursday 03/15/2012** | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **label** | **n** | **tps** | **median** | **mean** | **sdev** | **cv** | **p90** | **p95** | **p99** | **min** | **max** |
| **Concurrent Events** | **171699** | 143.20 | 5 | 6.21 | 3.66 | 0.59 | 11 | 13 | 19 | 0 | **33** |
| **Response Times** | **92984** | **77.55** | 11 | **74.97** | 228.82 | 3.05 | 88 | 617 | 1048 | 3 | 4060 |
| **Time Between Queries** | **92983** | 77.55 | 9 | 12.89 | 13.22 | **1.03** | 30 | 39 | 59 | 0 | 341 |

**Figure 5: Toolkit2 Demo Statistical Report Totals**

The gray cells in Figure 5 are of interest from a concurrency perspective as well as a test traffic quality orientation. The first data row reiterates two of the statistics listed in Figure 4, the total number of state transition timestamps processed, n = 171,699 and the maximum number of concurrent queries observed, max = 33. The second data row contains the query count, n = 92,984, the query rate, tps = 77.55 trans/sec, and response time statistics, e.g., mean = 74.97 milliseconds. The query count is the total number of data rows in the file processed.

The last row provides “Time Between Queries” statistics with a count value, n, that is one less than the query count because the numbers are the differences between sorted order query timestamps. The coefficient of variation, cv = sdev / mean, of the “Time Between Queries” metric is an indication of request independence, a key measure of offered traffic quality. The observed value of cv = 1.03 is very close to the ideal 1.00, indicating the load generator is mimicking real-world user request timing. For a detailed discussion of request independence and its impact on test traffic quality see [5.].

## 4.0 Queueing Model View of Concurrent Events

The Figure 1 load testing setup can be viewed as a queueing system with 200 traffic sources (virtual user threads) and 8 servers (CPU cores) where the GET/POST circles in the SUT represent nine concurrent events. If an application’s traffic flow characteristics match a queueing model’s mathematical assumptions, the SUT’s concurrency distribution can be approximated by the model’s state probabilities.

One key assumption many common queueing models possess, including those in the QueState repository, is the request independence property of the Toolkit2 demo. With this traffic flow characteristic in common, how well do the empirical state probabilities, “Prob”, of Figure 4 fit the appropriate queueing model’s state probability distribution? If there is a reasonable match, that model may be useful as a concurrency distribution estimator for applications anticipated to have the independent request property but in the early stages of design or development before measurements can be performed.

Two queueing models are chosen for this comparison, the details of which are contained in Appendix C. In Kendall Notation the models are:

1. M/M/c/N - Markovian arrivals and service times with c servers and fixed number of traffic sources (N)
2. M/M/c - Markovian arrivals and service with c servers and an infinite number of traffic sources

The Markovian property, the two M’s in the Kendall notation, imply arrivals occur independently (M) and service times are Negative-Exponentially distributed (M).

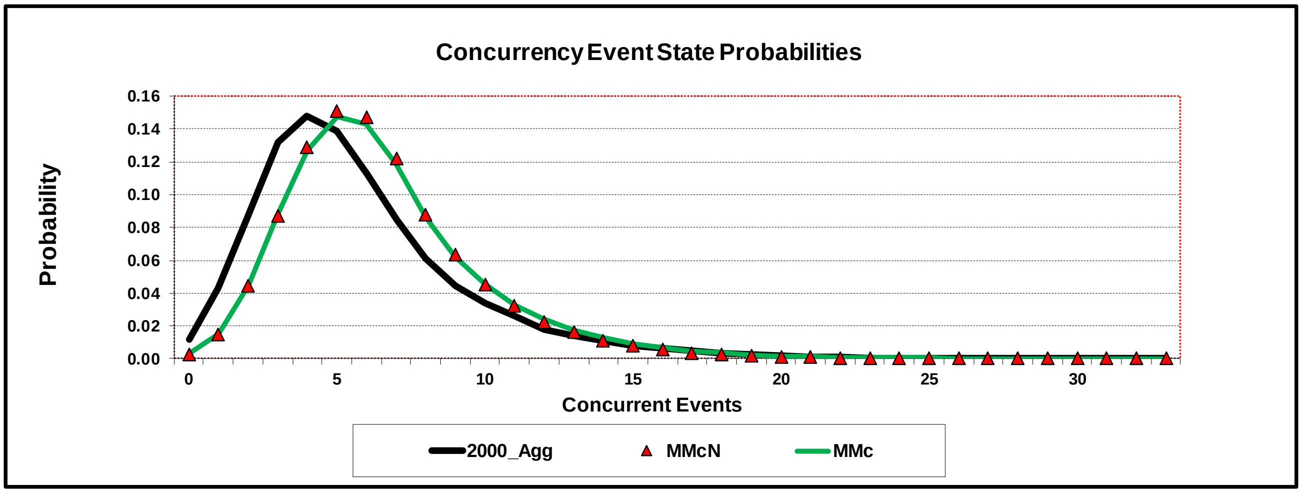
The M/M/c/N model accommodates the fact that the virtual user thread pool, i.e., users, is fixed and a thread in queue or service cannot be an arrival, whereas, the M/M/c model assumes a dynamically changing set of users. As will be illustrated, this model difference is superfluous for a group of user threads as large as 200.

Figure 6 lists the queueing model configuration parameters and their relationship to the information contained in previous figures. Figure1 identifies the number of servers, c = 8, and traffic sources, N = 200, while Figure 4 includes the mean number of concurrent events, λ/μ = 5.81, and Figure 5 provides the query arrival rate, λ = 77.55 tps. This information is entered into the two models to produce their QueState demo state probabilities. Appendix C contains GitHub repository execution logistics for these results.

| Queueing Model Input Parameters | | | | |
|---|---|---|---|---|
| **Parameter** | **Symbol** | **Value** | **Source** | **Comment** |
| **Servers** | c | 8 | Figure 1 | CPU Cores in SUT |
| **Sources** | N | 200 | Figure 1 | Load Generator Threads |
| **Arrival Rate** | λ | 77.55 | Figure 5 | Response Time Row - tps |
| **Service Rate** | μ | 13.34 | Figure 4 and Figure 5 | Calculated From λ and λ/μ |
| **Traffic** | λ/μ | 5.81 | Figure 4 | Mean Value Statistic |

**Figure 6: Queueing Model Input Parameters**

Figure 7 contains a comparison of the queueing model state probabilities (red triangles and green line) with the load test proportion of time in state empirical state probabilities (black solid line) where "Probability" is plotted as a function of "Concurrent Events".



| Table A | User Request Concurrency - 2000_Agg Test | | | | | |
|---|---|---|---|---|---|---|
| **Statistic** | **mean** | **sdev** | **var** | **max** | **p_sum** | **f_sum** |
| **Value** | 5.81 | 3.65 | 13.35 | 33 | 1.000000 | 171699 |

| Table B | Queueing Models | | | | | |
|---|---|---|---|---|---|---|
| **Param** | **Servers** | **Sources** | **λ/μ** | **λ** | **μ** | **p_sum** |
| **MMcN** | 8 | 200 | 5.81 | 77.55 | 13.34 | 0.999993 |
| **MMc** | 8 | ∞ | 5.81 | 77.55 | 13.34 | 0.999922 |

| Table C | State Probabilities | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **State** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** |
| **2000_Agg** | 0.01181 | 0.04314 | 0.08711 | 0.13177 | 0.14784 | 0.13866 | 0.11296 | 0.08505 | 0.06141 | 0.04481 | 0.03388 | 0.02629 | 0.01817 | 0.01442 | 0.01087 | 0.00780 | 0.00621 |
| **MMcN** | 0.00248 | 0.01482 | 0.04415 | 0.08723 | 0.12861 | 0.15093 | 0.14685 | 0.12184 | 0.08799 | 0.06322 | 0.04519 | 0.03213 | 0.02272 | 0.01599 | 0.01119 | 0.00779 | 0.00539 |
| **MMc** | 0.00266 | 0.01547 | 0.04496 | 0.08712 | 0.12661 | 0.14721 | 0.14263 | 0.11845 | 0.08607 | 0.06255 | 0.04545 | 0.03303 | 0.02400 | 0.01744 | 0.01267 | 0.00921 | 0.00669 |
| **State** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** | **29** | **30** | **31** | **32** | **33** |
| **2000_Agg** | 0.00506 | 0.00374 | 0.00252 | 0.00180 | 0.00140 | 0.00104 | 0.00074 | 0.00055 | 0.00034 | 0.00015 | 0.00020 | 0.00011 | 0.00004 | 0.00004 | 0.00005 | 0.00002 | 0.00001 |
| **MMcN** | 0.00371 | 0.00254 | 0.00173 | 0.00117 | 0.00079 | 0.00053 | 0.00035 | 0.00023 | 0.00015 | 0.00010 | 0.00007 | 0.00004 | 0.00003 | 0.00002 | 0.00001 | 0.00001 | 0.00000 |
| **MMc** | 0.00486 | 0.00353 | 0.00257 | 0.00187 | 0.00136 | 0.00099 | 0.00072 | 0.00052 | 0.00038 | 0.00027 | 0.00020 | 0.00015 | 0.00011 | 0.00008 | 0.00006 | 0.00004 | 0.00003 |

**Figure 7: State Probability Distribution For 2000_Agg Test and Two Queueing Models**

The graph is produced with the "Value" row of Figure 4 as Table A where the mean value statistic in that table is the queueing model offered load, λ/μ = 5.81, in Table B. The Figure 4 "Prob" data is listed in the two 2000_Agg rows of Table C and the queueing model state probabilities are enumerated in the MMcN and MMc rows of the same table. The MMcN and MMc rows of Table C are the state probability portion of model output for the first thirty-four states (0 – 33) which consume nearly all the probability, p_sum for M/M/c/N = .999993 and M/M/c = .999922.

A comparison of the 2000_Agg empirical results with the two sets of queueing model state probabilities leads to the following observations regarding the Figure 7 illustration:

1. The queueing model state probabilities are biased to the right of the 2000_Agg test run empirical state probabilities in the higher density section of the curve but fit well in the right-hand tail where the important maximums are listed. It is unclear why this bias exists but perhaps it's because both queueing models assume Negative-Exponentially distributed service times and SUT service times are likely to have a smaller variance.
2. The two queueing models (red diamonds and green line) have virtually identical state probabilities for all states shown, making them interchangeable as modeling tools when user populations are this large, i.e., 200 virtual user threads. This is important because M/M/c is a much easier model to implement than M/M/c/N. See Appendix C for the set of equations and list of assumptions associated with these two models.

The queueing model plots are certainly a close fit from a modeling perspective, making them a potentially useful tool for configuring internal system resources for applications anticipated to have the independent request property but still on the drawing board or in the early stages of development.

### 5.0 Summary

Load testing professionals are often asked to estimate how many concurrent requests the new application will support when there are no direct measurements available on the SUT to make that capacity determination. The approach taken here is to approximate the concurrent user request distribution with either data generated by the load tool or queueing models when test data is not available. It is hoped the Toolkit2 software combined with its Apache JMeter setup is of value when approximating concurrency with test data and the QueState queueing models are of use when preliminary estimation is required.

The approach taken to describe these two methods in detail beings with an overview of concurrency calculations as a means of dimensioning internal SUT resources. That overview is followed by the Section 2 graphical illustration of concurrent events residing in a load generation environment. This pictorial representation is enriched in the Section 3 load testing example taken from the Toolkit2 demo. That demo's input data is used as a vehicle for explaining the Toolkit2 concurrency calculation methodology with software specifics listed in Appendix A. This section also includes Toolkit2 demo summary results from a concurrency as well as traffic quality perspective with each of these orientations expanded upon in the Appendix B set of tables. In Section 4, the demo's traffic quality indicator points to the same user request pattern as the QueState models assume so a comparison is made between the load tool data and two of those eight models. The state probability equations and associated QueState model assumptions are listed in Appendix C.

Because the Toolkit2 software uses data gathered by the load generator rather than recorded in the SUT, caution should be used when interpreting the results. Long or erratic network latencies between the load generator and the SUT may skew the concurrency numbers since time in SUT includes network latency with this technique.

There are other queueing model programs on the internet which generate performance metrics, but few of them compute the state probabilities needed for concurrency estimation like the QueState programs. Although concurrency estimation is the primary application in this environment, these eight models are general purpose and can be used to analyze any situation where they are a reasonable fit.

The software contained in the web-generator-toolkit2 and QueState GitHub repositories is free, downloadable, and modifiable, with all code in the "bin" directories written in Perl. It is hoped performance analysts will take advantage of these repositories and share their experiences.

### Acknowledgment

This author would like to acknowledge Xiaosong Lou [9.] for sparking my interest in user request concurrency analysis and sharing the proportion of time in each state concurrency technique that I implemented in the Toolkit2 statistical analysis software.

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# Appendix A

## Concurrency Function Source Code
## (web_generator_toolkit2.pl)

```
sub
concurrency
{
  my($concurrency_ref) = @_;
  #########################################################
  # User Request Concurrency Analysis:                    #
  # - Technique Introduced To This Author By Xiaosong Lou #
  #########################################################
  # Variable Declarations Not Shown - See GitHub Repo     #
  # Variable Declarations - Initialized                   #
  #########################################################
  my ($currentEvents)=0;
  my ($msec_sum)=0;
  ################################
  # Read list                    #
  ################################
  foreach $record (@$concurrency_ref){
    ($beg,$rt) = split ('\,',$record);
    ################################
    # Create End Timestamp         #
    ################################
    $end = $beg + $rt;
    ##############################################
    # Run Counter For Duplicate Begin Timestamps #
    ##############################################
    if ($timestamps{$beg}){
      $timestamps{$beg}++;
    }
    else{
      $timestamps{$beg}=1;
    }
    ############################################
    # Run Counter For Duplicate End Timestamps #
    ############################################
    if ($timestamps{$end}){
      $timestamps{$end}--;
    }
    else{
      $timestamps{$end}=-1;
    }
  }
  ###############################################
  # Process timestamps in ascending order       #
  ###############################################
  foreach $timestamp (sort keys(%timestamps)){
    ###############################################
    # Calculate msec for last timestamp           #
    ###############################################
    if ($lastTimestamp){
      $msec = $timestamp-$lastTimestamp;
    }
    else{
      $msec = 0;
    }
```

# Appendix A - Continued

```
    ###############################################
    # Add msec to current histogram entry         #
    ###############################################
    if ($histogram{$currentEvents}){
      ($count,$ms) = split('\,',$histogram{$currentEvents});
      $count++;
      $ms+=$msec;
      $count_ms = join ',',$count,$ms;
      $histogram{$currentEvents} = $count_ms;
    }
    else{
      $count=1;
      $count_ms = join ',',$count,$msec;
      $histogram{$currentEvents} = $count_ms;
    }
    ###############################################
    # Put current event count on list             #
    ###############################################
    push @concurrentEvents,$currentEvents;
    ###############################################
    # Update counter variables                    #
    ###############################################
    $currentEvents += $timestamps{$timestamp};
    $lastTimestamp = $timestamp;
    $msec_sum += $msec;
  }
  ######################################################
  # Create state probability array                     #
  ######################################################
  foreach $pstate_val (sort keys(%histogram){
    ($count,$ms) = split('\,',$histogram{$pstate_val});
    if ($msec_sum){
      $prob = $ms/$msec_sum;
    }
    else{
      $prob = 0;
    }
    $count_prob = join ',',$count,$prob;
    $pstate[$pstate_val] = $count_prob;
  }
  ###############################################
  # Sort concurrent events list                 #
  ###############################################
  foreach $concurrentEvent (@concurrentEvents){
    $events = sprintf('%08d',$concurrentEvent);
    push @concurrentEvents_out,$events;
  }
  @concurrentEvents_out = sort @concurrentEvents_out;
  ######################################################
  # Assign unobserved state probabilities zero         #
  ######################################################
  for($i=0;$i<@pstate;$i++){
    if (!$pstate[$i]){
      $pstate[$i] = join ',',0,0;
    }
    $pstate[$i] = join ',',$i,$pstate[$i];
  }
  return(\@concurrentEvents_out,\@pstate);
}
```

# Appendix B

## Toolkit2 Demo Output Files

## (web-generator-toolkit2/demo)

### Figure 4: Concurrent Events

The GitHub location of the concurrent events for “Total” csv file produced by running the Toolkit2 demo.

1. Toolkit2 -> histograms -> concur -> demo_2000_AggRpt_120_1199_Total_concur.csv

### Figure 5: Full Reports:

The GitHub locations of the three full statistical reports produced by running the Toolkit2 demo.

1. Concurrent Events – Toolkit2 -> statistics –> demo_2000_AggRpt_120_1199_20120315_concur.csv
2. Response Times – Toolkit2 -> statistics –> demo_2000_AggRpt_120_1199_20120315_agg.csv
3. Time Between Queries - Toolkit2 -> statistics –> demo_2000_AggRpt_120_1199_20120315_arr.csv

1. **Concurrent Events**

| Concurrent Event Statistics (Count) - demo_2000_AggRpt_120_1199 Thursday 03/15/2012 | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **label** | **n** | **tps** | **median** | **mean** | **sdev** | **cv** | **p90** | **p95** | **p99** | **min** | **max** |
| **010_Home** | 47751 | 39.83 | 1 | 1.12 | 1.13 | 1.01 | 2 | 3 | 5 | 0 | 10 |
| **012_Home_jpg** | 48798 | 40.70 | 1 | 1.37 | 1.25 | 0.91 | 3 | 4 | 5 | 0 | 13 |
| **020_Dept** | 24719 | 20.62 | 1 | 0.85 | 0.85 | 1.00 | 2 | 2 | 4 | 0 | 7 |
| **022_Dept_jpg** | 24825 | 20.71 | 1 | 0.81 | 0.77 | 0.96 | 2 | 2 | 3 | 0 | 6 |
| **030_Demographics** | 24560 | 20.49 | 1 | 0.80 | 0.81 | 1.02 | 2 | 2 | 3 | 0 | 7 |
| **040_Statistics** | 12331 | 10.28 | 4 | 3.83 | 1.95 | 0.51 | 6 | 7 | 9 | 0 | 13 |
| **Total** | **171699** | 143.20 | 5 | 6.21 | 3.66 | 0.59 | 11 | 13 | 19 | 0 | **33** |

2. **Response Times**

| Aggregate Stats [Response Time(ms)] - demo_2000_AggRpt_120_1199 Thursday 03/15/2012 | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **label** | **n** | **tps** | **median** | **mean** | **sdev** | **cv** | **p90** | **p95** | **p99** | **min** | **max** |
| **010_Home** | 24384 | 20.34 | 9 | 31.60 | 161.33 | 5.11 | 20 | 62 | 516 | 5 | 3616 |
| **012_Home_jpg** | 24950 | 20.81 | 20 | 42.23 | 173.89 | 4.12 | 41 | 65 | 301 | 13 | 3864 |
| **020_Dept** | 12489 | 10.42 | 7 | 34.79 | 184.64 | 5.31 | 19 | 61 | 1412 | 4 | 3227 |
| **022_Dept_jpg** | 12533 | 10.45 | 5 | 30.07 | 183.20 | 6.09 | 14 | 33 | 1298 | 3 | 3430 |
| **030_Demographics** | 12424 | 10.36 | 7 | 30.04 | 164.54 | 5.48 | 17 | 53 | 1220 | 4 | 3217 |
| **040_Statistics** | 6204 | 5.17 | 622 | 638.70 | 171.31 | 0.27 | 674 | 768 | 1038 | 334 | 4060 |
| **Total** | **92984** | **77.55** | 11 | **74.97** | 228.82 | 3.05 | 88 | 617 | 1048 | 3 | 4060 |

3. **Time Between Queries**

| Inter-arrival Summary Statistics (ms) - demo_2000_AggRpt_120_1199 Thursday 03/15/2012 | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **label** | **n** | **tps** | **median** | **mean** | **sdev** | **cv** | **p90** | **p95** | **p99** | **min** | **max** |
| **010_Home** | 24383 | 20.34 | 34 | 49.17 | 49.55 | 1.01 | 113 | 148 | 228 | 0 | 533 |
| **012_Home_jpg** | 24949 | 20.81 | 33 | 48.06 | 48.66 | 1.01 | 111 | 145 | 227 | 0 | 527 |
| **020_Dept** | 12488 | 10.42 | 67 | 95.99 | 94.58 | 0.99 | 219 | 285 | 432 | 0 | 903 |
| **022_Dept_jpg** | 12532 | 10.45 | 66 | 95.67 | 95.74 | 1.00 | 221 | 288 | 437 | 0 | 871 |
| **030_Demographics** | 12423 | 10.36 | 67 | 96.51 | 97.87 | 1.01 | 224 | 295 | 450 | 0 | 990 |
| **040_Statistics** | 6203 | 5.17 | 133 | 193.24 | 196.31 | 1.02 | 445 | 579 | 910 | 0 | 1847 |
| **Total** | **92983** | 77.55 | 9 | 12.89 | 13.22 | **1.03** | 30 | 39 | 59 | 0 | 341 |

# Appendix C

## Queueing Model State Probabilities
## (QueState/demo)

### Figure 6: State Probability Input Parameters

The GitHub location of the state probability files produced by running the QueState demo.

1. QueState – MMcN -> MMcN_QueState_Results -> MMcN_2000_AggRpt_results_yyyymmddhhmmss.csv
2. QueState – MMc -> MMc_QueState_Results -> MMc_2000_AggRpt_results_yyyymmddhhmmss.csv

### Queueing Model Formulas

Below are the state probability formulas for the two queueing models. These equations are described in [6], [7], and [8] with the syntax from [6]. The symbols in parenthesis at the right of each variable definition are QueState repository terminology.

### M/M/c/N Model State Probabilities:

Key assumptions:
Source - the number of traffic sources is finite.
Arrival - arrivals occur quasi-randomly and have full access to all the servers.
Queue - blocked arrivals are delayed.
Service - service times are Negative-Exponentially distributed

Quasi-random arrivals imply no coercion exists between sources when making requests but the arrival rate changes as sources go in and out of the idle state, accounting for the fact that a source in queue or service cannot be an arrival. In this closed source system, the outside observer's and arriving customer's view differ but happily they are related in a simple way. The outside observer's n-Source state probability distribution, Eq. C1.1 and Eq. C1.2, is equal to the arriving customer's [n-1]-Source state probability distribution.

$$P_j[n] = \begin{cases} \binom{n}{j}\hat{a}^j P_0[n] & (j = 1,2,\ldots,s-1) \\ \frac{n!}{(n-j)!s!s^{j-s}}\hat{a}^j P_0[n] & (j = s, s+1, \ldots, n) \end{cases} \qquad (C1.1)$$

$Where\ P_0[n]\ is\ given$ by

$$P_0[n] = \left[\sum_{k=0}^{s-1}\binom{n}{k}\hat{a}^k + \sum_{k=s}^{n}\frac{n!}{(n-k)!\,s!\,s^{k-s}}\hat{a}^k\right]^{-1} \qquad (C1.2)$$

$Where$:
$n = number\ of\ traffic\ sources$ (N)
$\hat{a} = offered\ load\ per\ idle\ source\ (ai\_src)$
$s = number\ of\ servers$ (c)
$P_j[n] = state\ probability\ that\ j\ of\ the\ n\ sources\ are\ in\ the\ system\ (P[j])$
$P_0[n] = state\ probability\ that\ all\ n\ sources\ are\ idle\ (P[0])$

$The\ offered\ load\ per\ idle\ source, \hat{a}, is\ approximated\ by$:

$$\hat{a} = \frac{a_{src}}{1 - a_{src}} \qquad (C1.3)$$

$Where$:
$a_{src} = intended\ offered\ load\ per\ source\ (a\_src)$

$$a_{src} = \frac{\lambda/\mu}{n} \qquad (C1.4)$$

$Where$:
$\lambda = intended\ arrival\ rate\ (lamda)$
$\mu = service\ rate\ (mu)$

# Appendix C - Continued

## M/M/c Model State Probabilities:

Key assumptions:
  Source - the number of traffic sources is infinite.
  Arrival - arrivals occur randomly at a constant rate and have full access to all servers.
  Queue - blocked arrivals are delayed.
  Service - service times are Negative-Exponentially distributed.

Random arrivals imply no coercion exists between sources when making requests and arrivals are drawn from an infinite population at a constant rate. It is the well-known Poisson process where times between arrivals are Negative-Exponentially distributed and the number of arrivals in constant length intervals, are Poisson distributed. Unlike the closed source model, the outside observer and arriving customer share the same view of the system possessing the state probabilities shown in Eq. C1.5 and Eq. C1.6.

$$P_j = \begin{cases} \frac{a^j}{j!} P_0 & (j = 1,2, \dots, s-1) \\ \frac{a^j}{s! s^{j-s}} P_0 & (j = s, s+1, \dots) \end{cases} \qquad (C1.5)$$

$Where\ P_0\ is\ given\ by$

$$P_0 = \left[ \sum_{k=0}^{s-1} \frac{a^k}{k!} + \frac{a^s}{s!\,(1 - a/s)} \right]^{-1} \qquad (0 \le a < s) \qquad (C1.6)$$

$Where$:
$a = offered\ load\ (a)$
$s = number\ of\ servers$ (c)
$P_j = state\ probability\ that\ j\ customers\ are\ in\ system\ (P[j])$
$P_0 = state\ probability\ the\ system\ is\ empty\ (P[0])$

$$a = \lambda / \mu \qquad (C1.7)$$

$Where$:
$\lambda = arrival\ rate\ (lamda)$
$\mu = service\ rate\ (mu)$