Collatz Conjecture Proof for Special Integer Subsets and a Unified Criterion for Twin Prime Identification

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Abstract

This paper presents a proof of the Collatz conjecture for a specific subset of positive integers, those formed by multiplying a prime number "p" greater than three with an odd integer "u" derived using Fermat's little theorem. Additionally, we introduce a novel screening criterion for identifying candidate twin primes, extending our previous work linking twin primes (p and p+2) with the equation $2^{(p-2)} = pu+v$, where unique solutions for u and v are required. This unified criterion offers a promising approach to twin prime identification within a wider range of integers, further advancing research in this mathematical domain.

1 Introduction

The Collatz conjecture, a perplexing enigma in the realm of mathematics, has intrigued and perplexed mathematicians for decades. This conjecture posits that, regardless of the initial positive integer chosen, a sequence of operations involving division by 2 for even numbers and multiplication by 3 plus 1 for odd numbers will invariably lead to the value 1. While it might appear as a simple yet elusive puzzle, the Collatz conjecture remains one of the most enduring unsolved problems in mathematics, defying easy proof or disproof.

In this comprehensive exploration, we embark on a journey to unravel the mysteries of the Collatz conjecture. Our focus is on a distinctive subset of positive integers, a subset that emerges from the multiplication of a prime number "p," where p is greater than three, with another positive odd integer "u." These unique integers, intricately derived through the application of Fermat's little theorem, serve as the cornerstone of our investigation. Our mission: to provide a compelling and rigorous proof of the Collatz conjecture within the confines of this special class of integers.

This endeavor is not only a testament to the intellectual curiosity that propels mathematicians but also an effort to shed light on a specific subset of positive integers that bears a striking resemblance to prime numbers. Just as prime numbers hold a central role in number theory, these "pu" integers, arising from the intricate interplay of prime numbers and odd factors, form a fraction of positive integers that shares a certain mathematical kinship with primes.

As we venture deeper into the heart of the Collatz conjecture, we shall also draw connections to our previous research, which ventured into the realm of twin primes. Twin primes, pairs of primes differing by only two, have long tantalized mathematicians with their rarity and inherent mystery. In a prior study, we uncovered a fascinating relationship between twin primes (p and p+2) and a specific expression: $2^{(p-2)} = pu+v$ Importantly, this relationship demanded the existence of unique solutions for the variables u and v. This discovery marked a significant milestone in our exploration of twin primes, prompting us to extend our work.

Building upon this foundational research, we present a novel and unified criterion designed to facilitate the identification of candidate twin primes. This criterion represents an innovative approach that transcends the limitations of our earlier work, opening up new possibilities for the identification of twin primes within a broader spectrum of integers. By introducing this unified criterion, we aim to contribute to the ongoing journey of uncovering the secrets and patterns within the realm of twin primes, a pursuit that has fascinated mathematicians for centuries.

We delve into the intricacies of both the Collatz conjecture and our novel twin prime criterion, combining rigorous mathematical proofs with innovative thinking to expand our understanding of these fundamental mathematical phenomena. Our journey promises to offer fresh insights, challenging preconceived notions and bringing us one step closer to unraveling the mysteries that have captivated mathematicians for generations.[5][1][3][4][2]

2 Working for Prove 1

Collatz's conjecture can be straightforwardly confirmed for the prime numbers 2 and 3. However, when extending our analysis to prime numbers greater than 3, we turn to the power of Fermat's Little Theorem to provide the necessary insights.
Let
$2^p - 2 = pr$ where r is a positive integer
taking common 2 so
$2[2^{(p-1)} - 1] = pr$.....(i)
Since p is large prime it is odd and therefore it is of the form 2t+1 where t is a positive integer.
Let
p=2t+1 put in (i) where t is a positive integer.
Therefore left hand side can be written as

$$2[2^{(2t+1-1)} - 1] = pr$$
$$2[2^{(2t)} - 1] = pr$$

$$2[2^{(t)}-1][2^{(t)}+1]=pr$$

we taking L.H.S only

$$2[2^{(t)}-1].[2^{(t)}+1]$$

Since $2^t-1, 2^t, 2^t+1$ are three consecutive positive integers and 2^t is only divisible by 2 and therefore the product $(2^t-1)(2^t+1)$ must be divisible by 3. Therefore $2(2^{p-1}-1)$ must be divisible by 6.
Therefore pr must be divisible by 6. Since p is an odd prime greater than three,therefore a must be divisible by 6 and can be replaced as r=6u

$$2(2^{p-1}-1)=6pu$$

$$(2^{p-1}-1)=3pu$$

$$2^{p-1}=3pu+1$$

Since the Left hand side is even therefore 3pu must be odd, 3,p,u all odd.
Therefore

$$2^{p-1}=3pu+1$$

$$2^{p-1}=3(pu)+1$$

This resembles

$$2^{p-1}=3(oddinteger)+1$$

In accordance with the rules of Collatz's conjecture, when you start with an odd number in the form of "pu," the first step involves multiplying it by 3 and adding 1, resulting in an even number 2^{p-1}. This even number is a power of 2 and, therefore, will undergo "p-1" steps of the Collatz sequence, consisting of even numbers, until it reaches 1. Consequently, any number in the form of "pu" will require exactly "p" steps to reach the value of 1, according to the Collatz conjectu

3 Working for Prove 2

If p, p+2 are large twin primes numbers.
so

$$2^{(p-2)}=pu+v$$

(where u and v are unique solutions for any pair of twin primes)
Multiplying by 4,

$$2^p=4pu+4v$$

Subtracting 2 from both sides,

$$2^p-2=4pu+4v-2$$

Since p is prime, therefore it follows from Fermat's little theorem that $2^p - 2$ is divisible by p therefore

$$4v - 2 = p * w$$

(where W is an even integer, since p is large prime and therefore odd)
so

$$v = (p * w + 2)/4...........(i)$$

$$(2^{(p+2)} - 2) = x(p + 2)$$

where x=6v,(as shown)

$$v = (2^{(p+2)} - 2)/6(p + 2)$$

Therefore

$$v = (2^{(p+1)} - 1)/3(p + 2).....(ii)$$

From(i) and (ii) it follows that

$$(p * w + 2)/4 = (2^{(p+1)} - 1)/3(p + 2)$$
$$(p * w + 2) = 4(2^{(p+1)} - 1)/3(p + 2)$$
$$p * w = \frac{4(2^{(p+1)} - 1)}{3(p + 2)} - 2$$

final results

$$w = \frac{2(2^{(p+1)} - 3p - 8)}{3p(p + 2)}$$

Hence, when "p" and "p+2" denote two large twin prime numbers, the expression "$2^{(p+1)} - 3p - 8)$" is evenly divisible by three times the product of these twin primes. This particular criterion serves as a straightforward method for screening potential large twin prime candidates.

4 Conclusion

In conclusion, this paper has successfully provided a proof of the Collatz conjecture for a specific subset of positive integers, shedding light on the behavior of numbers formed by multiplying a prime number "p" greater than three with an odd integer "u" derived using Fermat's little theorem. Furthermore, we have introduced an innovative screening criterion for the identification of candidate twin primes, building upon our previous research that connects twin primes (p and p+2) with the equation $2^{(p-2)} = pu+v$, emphasizing the necessity of unique solutions for u and v. This unified criterion holds great promise in the realm of twin prime identification, extending its applicability to a broader range of integers and advancing the ongoing exploration of this intriguing mathematical field.

References

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