The Hyperbolic Sieve of Primes and Divisors

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The Hyperbolic Sieve of Primes and Divisors

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Abstract: This paper teaches us how to build a *Modular Lattice-Grid*. From this Modular Lattice-Grid, we introduce the *Hyperbolic Sieve of Primes and Composite Divisors*.
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2010 Mathematics Subject Classification: 11N35, 11N36, 11A05, 11A51.

1 Introduction

In this chapter, we will introduce a new Hyperbolic Sieve of Primes and Composite divisors. The basis of that Hyperbolic Sieve is the lattice-grid. The dots in the lattice-grid assume modular values. That is, the values of the lattice-grid dots will appear countless times and not finite times as in other sieves.

For this lattice-grid, we will give the name of Modular Lattice Grid.

In this way, it will be possible not only to sieve the Primes numbers, but it will also be possible to sieve through all the divisors of the Composite numbers.

For this sieve, we will give the name of Hyperbolic Sieve of Primes and Composite Divisors.

2 Constructing the Modular Lattice-Grid

Let's start constructing the rows.
The row 1 we fill with the periodic sequence A000012 Period 1: repeat [1,]; a(n) = 1 + (n mod 1).

There is an isomorphism with A000004 a(n) = n mod 1.

The row 2 we fill with the periodic sequence A000034 Period 2: repeat [1, 2]; a(n) = 1 + (n mod 2).

There is an isomorphism with A000035 a(n) = n mod 2.

The row 3 we fill with the periodic sequence A010882 Period 3: repeat [1,2,3]; a(n) = 1 + (n mod 3).

There is an isomorphism with A010872 a(n) = n mod 3.

The row 4 we fill with the periodic sequence A010883 Period 4: repeat [1,2,3,4]; a(n) = 1 + (n mod 4).

There is an isomorphism with A010873 a(n) = n mod 4.

The row 5 we fill with the periodic sequence A010884 Period 5: repeat [1,2,3,4,5]; a(n) = 1 + (n mod 5).

There is an isomorphism with A010874 a(n) = n mod 5.

The row 6 we fill with the periodic sequence A010885 Period 6: repeat [1,2,3,4,5,6]; a(n) = 1 + (n mod 6).

There is an isomorphism with A010875 a(n) = n mod 6.

The row 7 we fill with the periodic sequence A010886 Period 7: repeat [1,2,3,4,5,6,7]; a(n) = 1 + (n mod 7).

There is an isomorphism with A010876 a(n) = n mod 7.

The row 8 we fill with the periodic sequence A010887 Period 8: repeat [1,2,3,4,5,6,7,8]; a(n) = 1 + (n mod 8).

There is an isomorphism with A010877 a(n) = n mod 8.

The row 9 we fill with the periodic sequence A177274 Period 9: repeat [1,2,3,4,5,6,7,8,9]; a(n) = 1 + (n mod 9).

There is an isomorphism with A010878 a(n) = n mod 9.

The row 10 we fill with the periodic sequence A010889 Period 10: repeat [1,2,3,4,5,6,7,8,9,10]; a(n) = 1 + (n mod 10).

There is an isomorphism with A010879 a(n) = n mod 10.

And so on…

As a reference, all these periodic sequences can be summarized in a comparative table where the fractions that generate the equivalent recurring are analyzed:

OEIS	a(n) = 1 + (n mod k)	Divisions by Rep(9)	Divisions by Rep(3)	Divisions by Repunit
A000012	a(n) = 1 + (n mod 1)	$\frac{1}{9} = \frac{0,\overline{3}}{3} = \frac{0,\overline{1}}{1} = 0,\overline{1}$	$\frac{4}{3} = 1,\overline{3}$	$\frac{1}{1} = 1,\overline{0}$
A000034	a(n) = 1 + (n mod 2)	$\frac{12}{99} = 0,\overline{12}$	$\frac{4}{33} = 0,\overline{12}$	$\frac{1}{11} = 0,\overline{09}$
A010882	a(n) = 1 + (n mod 3)	$\frac{123}{999} = 0,\overline{123}$	$\frac{41}{333} = 0,\overline{123}$	$\frac{13}{111} = 0,\overline{117}$
A010883	a(n) = 1 + (n mod 4)	$\frac{1234}{9999} = 0,\overline{1234}$	$\frac{411}{3333} = 0,\overline{1233}$	$\frac{137}{1111} = 0,\overline{1233}$
A010884	a(n) = 1 + (n mod 5)	$\frac{12345}{99999} = 0,\overline{12345}$	$\frac{4115}{33333} = 0,\overline{12345}$	$\frac{1371}{11111} = 0,\overline{12339}$
A010885	a(n) = 1 + (n mod 6)	$\frac{123456}{999999} = 0,\overline{123456}$	$\frac{41152}{333333} = 0,\overline{123456}$	$\frac{13717}{111111} = 0,\overline{123453}$
A010886	a(n) = 1 + (n mod 7)	$\frac{1234567}{9999999} = 0,\overline{1234567}$	$\frac{411522}{3333333} = 0,\overline{1235466}$	$\frac{137174}{1111111} = 0,\overline{1235466}$
A010887	a(n) = 1 + (n mod 8)	$\frac{12345678}{99999999} = 0,\overline{12345678}$	$\frac{4115226}{33333333} = 0,\overline{12345678}$	$\frac{1371742}{11111111} = 0,\overline{12345678}$
A177274	a(n) = 1 + (n mod 9)	$\frac{123456789}{999999999} = 0,\overline{123456789}$	$\frac{41152263}{333333333} = 0,\overline{123456789}$	$\frac{13717421}{111111111} = 0,\overline{123456789}$
A010889	a(n) = 1 + (n mod 10)	$\frac{12345678910}{99999999999} = 0,\overline{12345678910}$	$\frac{4115226303}{33333333333} = 0,\overline{12345678909}$	$\frac{1371742101}{11111111111} = 0,\overline{12345678909}$

OEIS	a(n) = n mod k	Divisions by Rep(9)	Divisions by Rep(3)	Divisions by Repunit
A000004	a(n) = n mod 1	$\frac{0}{9} = 0,\overline{0}$	$\frac{0,\overline{0}}{3} = 0,\overline{0}$	$\frac{0,\overline{0}}{1} = 0,\overline{0}$
A000035	a(n) = n mod 2	$\frac{1}{99} = 0,\overline{01}$	$\frac{0,\overline{3}}{33} = 0,\overline{01}$	$\frac{0,\overline{1}}{11} = 0,\overline{01}$
A010872	a(n) = n mod 3	$\frac{12}{999} = 0,\overline{012}$	$\frac{4}{333} = 0,\overline{012}$	$\frac{1}{111} = 0,\overline{009}$; $\frac{2}{111} = 0,\overline{018}$
A010873	a(n) = n mod 4	$\frac{123}{9999} = 0,\overline{0123}$	$\frac{41}{3333} = 0,\overline{0123}$	$\frac{13}{1111} = 0,\overline{0117}$; $\frac{14}{1111} = 0,\overline{0126}$
A010874	a(n) = n mod 5	$\frac{1234}{99999} = 0,\overline{01234}$	$\frac{411}{333333} = 0,\overline{01233}$	$\frac{137}{11111} = 0,\overline{01233}$
A010875	a(n) = n mod 6	$\frac{12345}{999999} = 0,\overline{012345}$	$\frac{4115}{333333} = 0,\overline{012345}$	$\frac{1371}{111111} = 0,\overline{012339}$
A010876	a(n) = n mod 7	$\frac{123456}{9999999} = 0,\overline{0123456}$	$\frac{41152}{3333333} = 0,\overline{0123456}$	$\frac{13717}{1111111} = 0,\overline{0123453}$
A010877	a(n) = n mod 8	$\frac{1234567}{99999999} = 0,\overline{01234567}$	$\frac{411522}{33333333} = 0,\overline{01234566}$	$\frac{137174}{11111111} = 0,\overline{01234566}$
A010878	a(n) = n mod 9	$\frac{12345678}{999999999} = 0,\overline{012345678}$	$\frac{4115226}{333333333} = 0,\overline{012345678}$	$\frac{1371742}{111111111} = 0,\overline{012345678}$
A010879	a(n) = n mod 10	$\frac{123456789}{9999999999} = 0,\overline{0123456789}$	$\frac{41152263}{3333333333} = 0,\overline{0123456789}$	$\frac{13717421}{1111111111} = 0,\overline{0123456789}$

Table 1. representation of Modular Lattice Grid sequences by recurring

2.1 Representation of Modular Lattice-Grid in XY-plane

Then, we get what we will call modular lattice-grid:

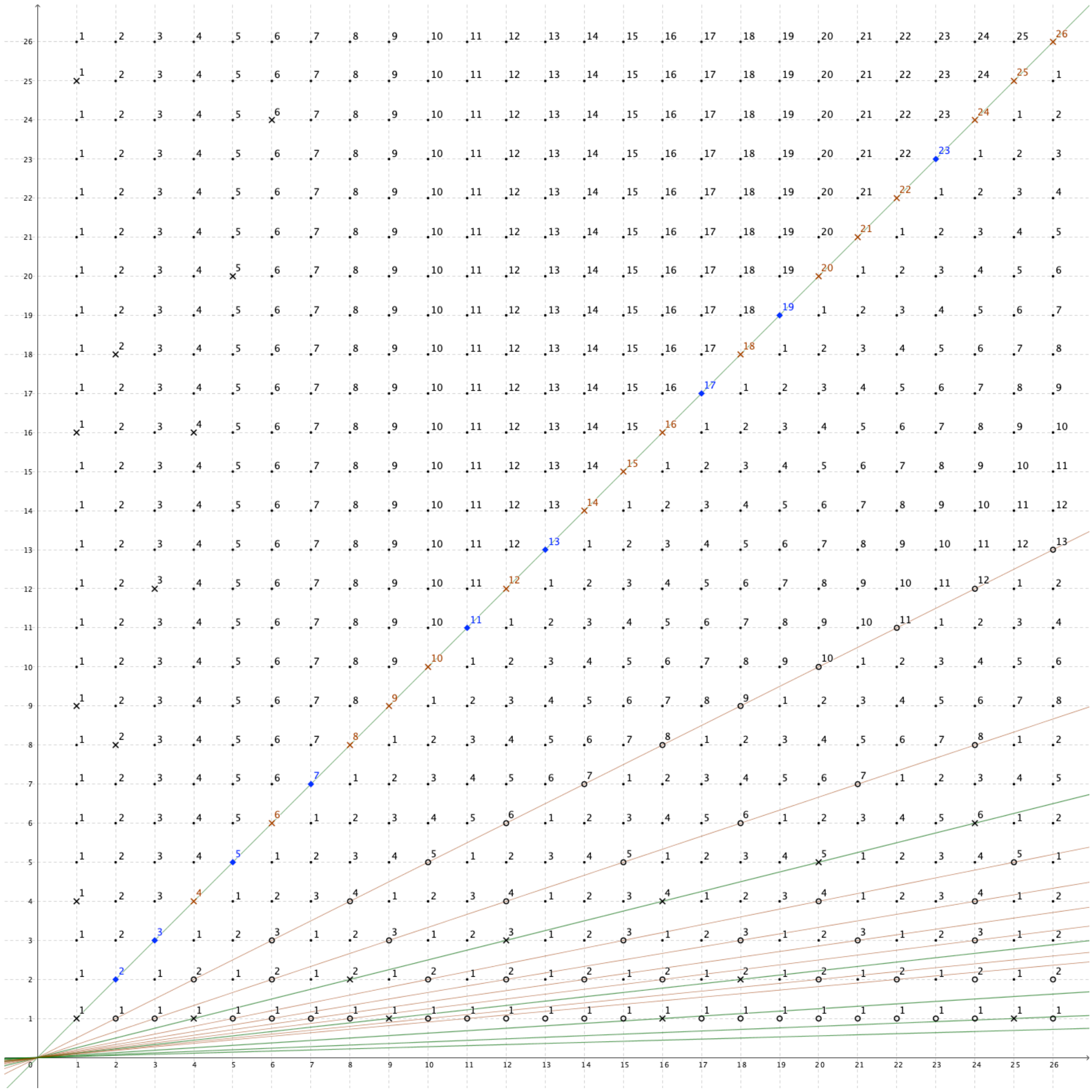

Figure 1. The Modular Lattice-Grid in XY-plane

In each row y, the values of their dots are repeated directly proportional to the value of y times. This linear and periodic repeat is what generates the modular property of this lattice-grid.

Thus, it is possible to draw infinitely many lines of the form $x = ny$ separating all possible modularity's.

See the enlarged figure showing the XY coordinates of each dot.

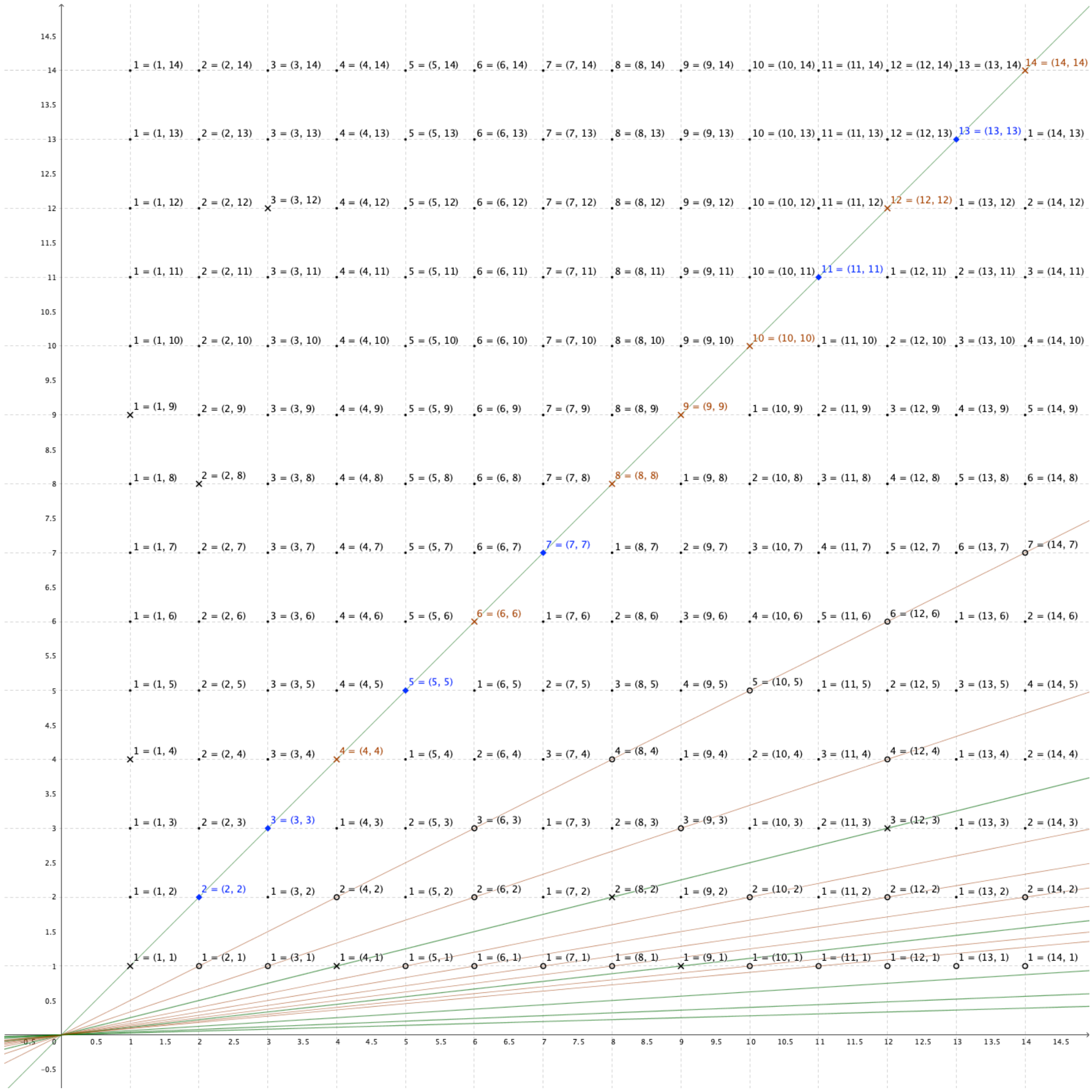

Figure 2. The divisor's lines in Modular Lattice-Grid

Note that the value of each dot is fully related to its coordinates.

For example,

- The dot $(x, y) = (7, 5)$ has value 2 because of $7 \; mod \; 5 = 2$.
- The dot $(x, y) = (5, 7)$ has value 5 because of $5 \; mod \; 7 = 5$.
- The dot $(x, y) = (12, 10)$ has value 2 because of $12 \; mod \; 10 = 2$.

That is, all dots (x, y) of our lattice-grid have a value of the form $z = x \; mod \; y$, except when $y|x$. When $y|x$, then $z = y$.

For example,

- The dot $(x, y) = (4, 4)$ has value 4 because of $4 \; mod \; 4 = 0$ and $y = 4$.
- The dot $(x, y) = (12, 4)$ has value 4 because of $12 \; mod \; 4 = 0$ and $y = 4$.

2.2 Primality by Geometry

As a consequence of this construction, we can obtain a primality test using equivalent triangles geometry.

Any composite number will have one of the slope lines $\left(\frac{1}{integer}\right)$ passing through one of its dots located in its vertical column.

Prime number columns do not have dots with slope lines $\left(\frac{1}{integer}\right)$ passing over them.

For example, 6 is not a prime because, besides lines $y = \frac{1}{1}x$ and $y = \frac{1}{6}x$, it has two lines $y = \frac{1}{3}x$ and $y = \frac{1}{2}x$ passing over the dots 2 and 3 respectively in column 6.

In column 5, 5 is a prime because there are only lines $y = \frac{1}{1}x$ and $y = \frac{1}{5}x$ passing through dots in column 5.

2.3 Study of Modular Lattice-Grid Columns

To be more didactic in the study of the columns, let's put the values obtained from each dot in a table.

For any x mod y = 0, then A(x,y) = y. There is an isomorphism with A(x,y) = x mod y.

26	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	1
24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	1	2
23	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	1	2	3
22	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	1	2	3	4
21	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	1	2	3	4	5
20	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	1	2	3	4	5	6
19	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	1	2	3	4	5	6	7
18	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	1	2	3	4	5	6	7	8
17	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	1	2	3	4	5	6	7	8	9
16	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	1	2	3	4	5	6	7	8	9	10
15	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	1	2	3	4	5	6	7	8	9	10	11
14	1	2	3	4	5	6	7	8	9	10	11	12	13	14	1	2	3	4	5	6	7	8	9	10	11	12
13	1	2	3	4	5	6	7	8	9	10	11	12	13	1	2	3	4	5	6	7	8	9	10	11	12	13
12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2
11	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4
10	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6
9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8
8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2
7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5
6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2
5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1
4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2
3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2
2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

26	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
25	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	1	2
24	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	1	2	3	4
23	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	1	2	3	4	5	6
22	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	1	2	3	4	5	6	7	8
21	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	1	2	3	4	5	6	7	8	9	10
20	7	8	9	10	11	12	13	14	15	16	17	18	19	20	1	2	3	4	5	6	7	8	9	10	11	12
19	8	9	10	11	12	13	14	15	16	17	18	19	1	2	3	4	5	6	7	8	9	10	11	12	13	14
18	9	10	11	12	13	14	15	16	17	18	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
17	10	11	12	13	14	15	16	17	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	1
16	11	12	13	14	15	16	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	1	2	3	4
15	12	13	14	15	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	1	2	3	4	5	6	7
14	13	14	1	2	3	4	5	6	7	8	9	10	11	12	13	14	1	2	3	4	5	6	7	8	9	10
13	1	2	3	4	5	6	7	8	9	10	11	12	13	1	2	3	4	5	6	7	8	9	10	11	12	13
12	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4
11	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8
10	7	8	9	10	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	1	2
9	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7
8	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4
7	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3
6	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4
5	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2
4	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
3	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1
2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52

Figure 3. The Modular Lattice-Grid in table mode

Each of the terms in our table varies according to the coordinate in the XY plane. Therefore, each dot is a function of type A(x,y).

All dots in light red have a line $y = \left(\frac{1}{integer}\right)x$ passing to it, and the value of the dot is the column of the dot divided by the integer.

As we are seeing the sequences in the vertical columns, then let's express each term of the vertical sequences as being $a(y)$. This is to remember that in each column x, each term varies depending on the value of y.

The law of formation of this table is given by the following algorithm:

Each vertical sequence starts at row $y = 1$.

Each term in the sequence has the value given by $a(y) = x\ mod\ y$. But if $x\ mod\ y = 0$, then $a(y) = y$.

See that our table above has a total isomorphism with the table below where $a(y) = x\ mod\ y$.

A(x,y) = x mod y

26	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	0
25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	0	1
24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	0	1	2
23	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	0	1	2	3
22	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	0	1	2	3	4
21	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	0	1	2	3	4	5
20	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0	1	2	3	4	5	6
19	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	0	1	2	3	4	5	6	7
18	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	0	1	2	3	4	5	6	7	8
17	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	0	1	2	3	4	5	6	7	8	9
16	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	0	1	2	3	4	5	6	7	8	9	10
15	1	2	3	4	5	6	7	8	9	10	11	12	13	14	0	1	2	3	4	5	6	7	8	9	10	11
14	1	2	3	4	5	6	7	8	9	10	11	12	13	0	1	2	3	4	5	6	7	8	9	10	11	12
13	1	2	3	4	5	6	7	8	9	10	11	12	0	1	2	3	4	5	6	7	8	9	10	11	12	0
12	1	2	3	4	5	6	7	8	9	10	11	0	1	2	3	4	5	6	7	8	9	10	11	0	1	2
11	1	2	3	4	5	6	7	8	9	10	0	1	2	3	4	5	6	7	8	9	10	0	1	2	3	4
10	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6
9	1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7	8
8	1	2	3	4	5	6	7	0	1	2	3	4	5	6	7	0	1	2	3	4	5	6	7	0	1	2
7	1	2	3	4	5	6	0	1	2	3	4	5	6	0	1	2	3	4	5	6	0	1	2	3	4	5
6	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2
5	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1
4	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2
3	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2
2	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

26	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	0
25	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	0	1	2
24	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	0	1	2	3	4
23	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	0	1	2	3	4	5	6
22	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	0	1	2	3	4	5	6	7	8
21	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	0	1	2	3	4	5	6	7	8	9	10
20	7	8	9	10	11	12	13	14	15	16	17	18	19	0	1	2	3	4	5	6	7	8	9	10	11	12
19	8	9	10	11	12	13	14	15	16	17	18	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
18	9	10	11	12	13	14	15	16	17	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
17	10	11	12	13	14	15	16	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	0	1
16	11	12	13	14	15	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	0	1	2	3	4
15	12	13	14	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	0	1	2	3	4	5	6	7
14	13	0	1	2	3	4	5	6	7	8	9	10	11	12	13	0	1	2	3	4	5	6	7	8	9	10
13	1	2	3	4	5	6	7	8	9	10	11	12	0	1	2	3	4	5	6	7	8	9	10	11	12	0
12	3	4	5	6	7	8	9	10	11	0	1	2	3	4	5	6	7	8	9	10	11	0	1	2	3	4
11	5	6	7	8	9	10	0	1	2	3	4	5	6	7	8	9	10	0	1	2	3	4	5	6	7	8
10	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2
9	0	1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7
8	3	4	5	6	7	0	1	2	3	4	5	6	7	0	1	2	3	4	5	6	7	0	1	2	3	4
7	6	0	1	2	3	4	5	6	0	1	2	3	4	5	6	0	1	2	3	4	5	6	0	1	2	3
6	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4
5	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2
4	3	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3	0
3	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1
2	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52

Figure 4. The Modular Lattice-Grid in isomorphic table mode

Consequently, each of the Zeros resultant from $x\ mod\ y = 0$ shows us the value y in the first table as being one of the divisors of x.

For example:

Column 10 in the second table we have the sequence A324471 $a(y) = 10\ mod\ y$ as being

{0, 0, 1, 2, 0, 4, 3, 2, 1, 0, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, …}.

There are 4 Zeroes in the whole sequence. So, number 10 has 4 divisors in its positions.

Column 10 in the first table we have the sequence AXXXXXXX $a(y) = 10\ mod\ y\ if\ 10\ mod\ y = 0\ then\ \ a(y) = y$. The sequence as being

{1, 2, 1, 2, 5, 4, 3, 2, 1, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, …}.

The correspondent positions give us the divisors of 10 as being (1,2,5,10).

This way of thinking substituting the zero in modular arithmetic by its equivalent non-zero integers is not new. For example, it is much studied in the digital root matter. Digital root arithmetic has specific isomorphism with modular arithmetic only for (mod 9).

Mod 9 works with results from the set (0,1,2,3,4,5,6,7,8) and digital root works with results from the set (1,2,3,4,5,6,7,8,9).

In the present study, we are generalizing the isomorphism to any set of modular arithmetic.

See the correspondence between the sequences between the two tables from Integer 1 until 20:

Integer	operation	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
1	Arith Modular	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	Isomorphism	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	Arith Modular	0	0	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	Isomorphism	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	Arith Modular	0	1	0	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
3	Isomorphism	1	1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4	Arith Modular	0	0	1	0	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4	Isomorphism	1	2	1	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
5	Arith Modular	0	1	2	1	0	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
5	Isomorphism	1	1	2	1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
6	Arith Modular	0	0	0	2	1	0	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	Isomorphism	1	2	3	2	1	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
7	Arith Modular	0	1	1	3	2	1	0	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
7	Isomorphism	1	1	1	3	2	1	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
8	Arith Modular	0	0	2	0	3	2	1	0	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
8	Isomorphism	1	2	2	4	3	2	1	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
9	Arith Modular	0	1	0	1	4	3	2	1	0	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
9	Isomorphism	1	1	3	1	4	3	2	1	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
10	Arith Modular	0	0	1	2	0	4	3	2	1	0	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
10	Isomorphism	1	2	1	2	5	4	3	2	1	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
11	Arith Modular	0	1	2	3	1	5	4	3	2	1	0	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
11	Isomorphism	1	1	2	3	1	5	4	3	2	1	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
12	Arith Modular	0	0	0	0	2	0	5	4	3	2	1	0	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12
12	Isomorphism	1	2	3	4	2	6	5	4	3	2	1	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12
13	Arith Modular	0	1	1	1	3	1	6	5	4	3	2	1	0	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13
13	Isomorphism	1	1	1	1	3	1	6	5	4	3	2	1	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13
14	Arith Modular	0	0	2	2	4	2	0	6	5	4	3	2	1	0	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
14	Isomorphism	1	2	2	2	4	2	7	6	5	4	3	2	1	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
15	Arith Modular	0	1	0	3	0	3	1	7	6	5	4	3	2	1	0	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
15	Isomorphism	1	1	3	3	5	3	1	7	6	5	4	3	2	1	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
16	Arith Modular	0	0	1	0	1	4	2	0	7	6	5	4	3	2	1	0	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
16	Isomorphism	1	2	1	4	1	4	2	8	7	6	5	4	3	2	1	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
17	Arith Modular	0	1	2	1	2	5	3	1	8	7	6	5	4	3	2	1	0	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17
17	Isomorphism	1	1	2	1	2	5	3	1	8	7	6	5	4	3	2	1	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17
18	Arith Modular	0	0	0	2	3	0	4	2	0	8	7	6	5	4	3	2	1	0	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18
18	Isomorphism	1	2	3	2	3	6	4	2	9	8	7	6	5	4	3	2	1	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18
19	Arith Modular	0	1	1	3	4	1	5	3	1	9	8	7	6	5	4	3	2	1	0	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19
19	Isomorphism	1	1	1	3	4	1	5	3	1	9	8	7	6	5	4	3	2	1	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19
20	Arith Modular	0	0	2	0	0	2	6	4	2	0	9	8	7	6	5	4	3	2	1	0	20	20	20	20	20	20	20	20	20	20	20	20	20	20
20	Isomorphism	1	2	2	4	5	2	6	4	2	10	9	8	7	6	5	4	3	2	1	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20

Figure 5. Table of Modular Lattice Grid columns in the 1st quadrant as modular arithmetic and isomorphism.

The number of divisors for each Integer x is given by the number of zeroes in its vertical sequence.

The number of zeroes is given by the sequence A000005 d(x) (also called tau(x) or sigma_0(x)), the number of divisors of x.

Starting from Integer $x = 1$, the sequence of the number of divisors are: {1, 2, 2, 3, 2, 4, 2, 4, 3, 4, 2, 6, 2, 4, 4, 5, 2, 6, 2, 6, 4, 4, 2, 8, 3, 4, 4, 6, 2, 8, 2, 6, 4, 4, 4, 9, 2, 4, 4, 8, 2, 8, 2, 6, 6, 4, 2, 10, 3, 6, 4, 6, 2, 8, 4, 8, 4, 4, 2, 12, 2, 4, 6, 7, 4, 8, 2, 6, 4, 8, 2, 12, 2, 4, 6, 6, 4, 8, 2, 10, 5, 4, 2, 12, 4, 4, 4, 8, 2, 12, 4, 6, 4, 4, 4, 12, 2, 6, 6, 9, 2, 8, 2, 8, …}.

2.4 Conclusions

1. The assumed value $a(y)$ of each dot (x, y) is given by
   a. For $x \bmod y \neq 0$, then $a(y) = x \bmod y$.
   b. For $x \bmod y = 0$, then $a(y) = y$.

2. The lines passing through dots $x \bmod y = 0$, reveal the divisors of x as being $a(y) = y$.

3. In each triangle, all formed by infinite numbers limited by the lines where $x \bmod y = 0$ (or where $y \mid x$), we have the same sequences, but an offset on the X-axis.

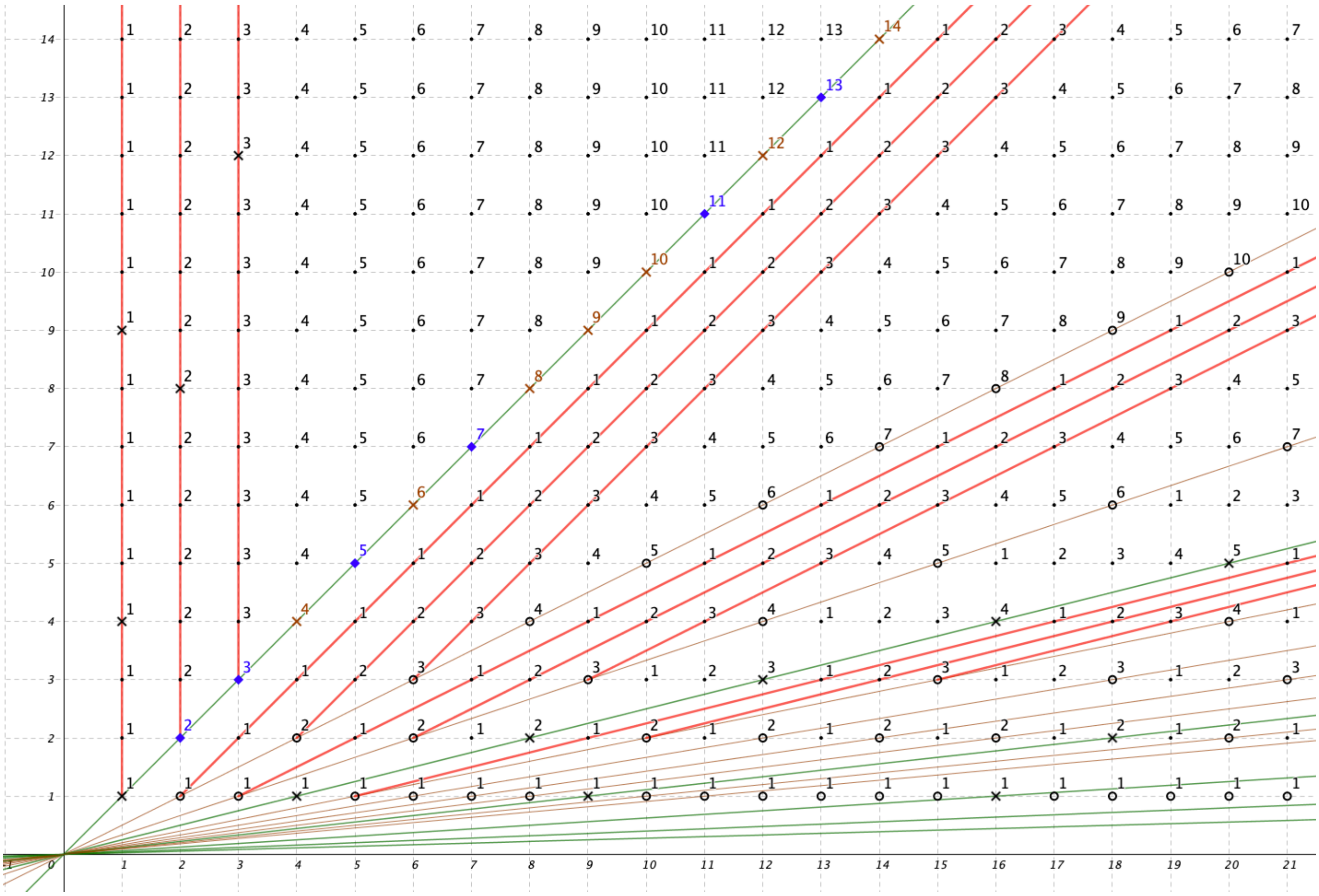

Figure 6. Infinitely many triangles formed by divisors lines. Each triangle has the same elements.

So, as we'll see next, we can place the zeros on the X and Y axes and expand our table to the negative sides, as well as making the rotations. But because it is a modular lattice-grid, here the rotation generates itself back. It's like a fractal because the values are all modular.

2.5 Study of Modular Lattice-Grid in Negative Rows and Columns

See below the table of the dots from $x \ mod \ y$, for any position in the XY-plane.

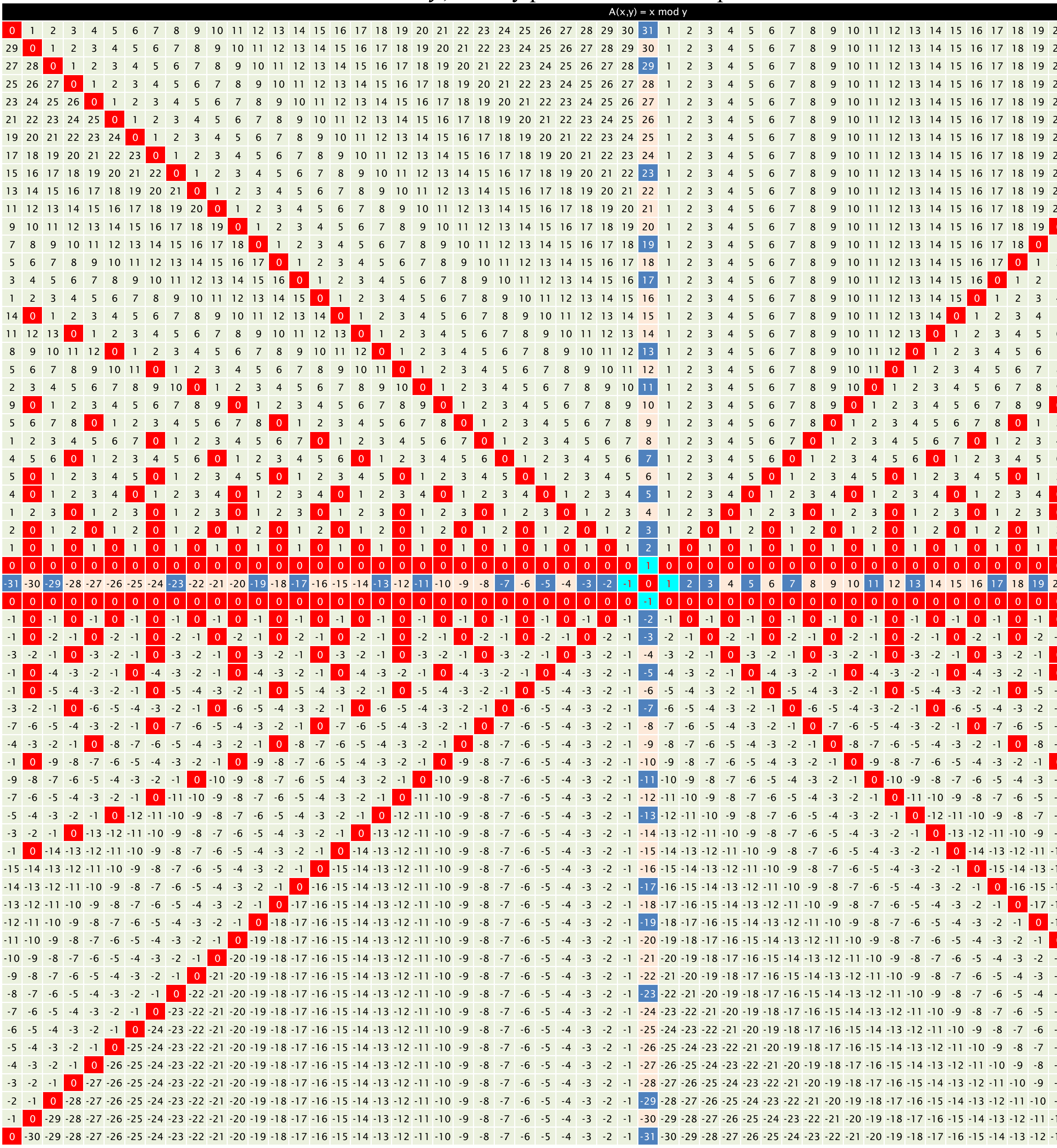

Figure 7. Each element has a value given by $A(x, y) = x \ mod \ y$.

Now, substituting the Zeroes values by the row y value, we get this isomorphic table:

Here, for any x mod y = 0, then A(x,y) = y. There is an isomorphism with A(x,y) = x mod y.

31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14
29	30	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	1	2	3	4	5	6	7	8	9	10	11	12	13	14
27	28	29	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	1	2	3	4	5	6	7	8	9	10	11	12	13	14
25	26	27	28	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	1	2	3	4	5	6	7	8	9	10	11	12	13	14
23	24	25	26	27	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	1	2	3	4	5	6	7	8	9	10	11	12	13	14
21	22	23	24	25	26	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	1	2	3	4	5	6	7	8	9	10	11	12	13	14
19	20	21	22	23	24	25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	1	2	3	4	5	6	7	8	9	10	11	12	13	14
17	18	19	20	21	22	23	24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	1	2	3	4	5	6	7	8	9	10	11	12	13	14
15	16	17	18	19	20	21	22	23	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	1	2	3	4	5	6	7	8	9	10	11	12	13	14
13	14	15	16	17	18	19	20	21	22	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	1	2	3	4	5	6	7	8	9	10	11	12	13	14
11	12	13	14	15	16	17	18	19	20	21	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	1	2	3	4	5	6	7	8	9	10	11	12	13	14
9	10	11	12	13	14	15	16	17	18	19	20	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	8	9	10	11	12	13	14	15	16	17	18	19	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	1	2	3	4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15	16	17	18	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	1	2	3	4	5	6	7	8	9	10	11	12	13	14
14	15	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	1	2	3	4	5	6	7	8	9	10	11	12	13	14
11	12	13	14	1	2	3	4	5	6	7	8	9	10	11	12	13	14	1	2	3	4	5	6	7	8	9	10	11	12	13	14	1	2	3	4	5	6	7	8	9	10	11	12	13	14
8	9	10	11	12	13	1	2	3	4	5	6	7	8	9	10	11	12	13	1	2	3	4	5	6	7	8	9	10	11	12	13	1	2	3	4	5	6	7	8	9	10	11	12	13	1
5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2
2	3	4	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8	9	10	11	1	2	3
9	10	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	1	2	3	4
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-1	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-10	-9	-8	-7	-6
-9	-8	-7	-6	-5	-4	-3	-2	-1	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-11	-10	-9	-8
-7	-6	-5	-4	-3	-2	-1	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-12	-11	-10
-5	-4	-3	-2	-1	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-13	-12
-3	-2	-1	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-14
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-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3
-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4
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-7	-6	-5	-4	-3	-2	-1	-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10
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-5	-4	-3	-2	-1	-26	-25	-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-26	-25	-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12
-4	-3	-2	-1	-27	-26	-25	-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-27	-26	-25	-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13
-3	-2	-1	-28	-27	-26	-25	-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-28	-27	-26	-25	-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14
-2	-1	-29	-28	-27	-26	-25	-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-29	-28	-27	-26	-25	-24	-23	-22	-21	-20	-19	-18	-17	-16	-15
-1	-30	-29	-28	-27	-26	-25	-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	-30	-29	-28	-27	-26	-25	-24	-23	-22	-21	-20	-19	-18	-17	-16
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Figure 8. Each element has a value given by $A(x,y) = x \ mod \ y$. If $x \ mod \ y = 0$ then, $A(x,y) = y$.

2.6 Sequences set in the 2nd quadrant

From the 2 tables above, we can now see the column’s sequences in the 2nd quadrant. Note that now we are getting sequences result from negative integers in modular arithmetic:

Integer	operation	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
-1	Arith Modular	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
-1	Isomorphism	1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
-2	Arith Modular	0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
-2	Isomorphism	1	2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
-3	Arith Modular	0	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
-3	Isomorphism	1	1	3	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
-4	Arith Modular	0	0	2	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
-4	Isomorphism	1	2	2	4	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
-5	Arith Modular	0	1	1	3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
-5	Isomorphism	1	1	1	3	5	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
-6	Arith Modular	0	0	0	2	4	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
-6	Isomorphism	1	2	3	2	4	6	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
-7	Arith Modular	0	1	2	1	3	5	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
-7	Isomorphism	1	1	2	1	3	5	7	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
-8	Arith Modular	0	0	1	0	2	4	6	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
-8	Isomorphism	1	2	1	4	2	4	6	8	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
-9	Arith Modular	0	1	0	3	1	3	5	7	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
-9	Isomorphism	1	1	3	3	1	3	5	7	9	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
-10	Arith Modular	0	0	2	2	0	2	4	6	8	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
-10	Isomorphism	1	2	2	2	5	2	4	6	8	10	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
-11	Arith Modular	0	1	1	1	4	1	3	5	7	9	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
-11	Isomorphism	1	1	1	1	4	1	3	5	7	9	11	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
-12	Arith Modular	0	0	0	0	3	0	2	4	6	8	10	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
-12	Isomorphism	1	2	3	4	3	6	2	4	6	8	10	12	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
-13	Arith Modular	0	1	2	3	2	5	1	3	5	7	9	11	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
-13	Isomorphism	1	1	2	3	2	5	1	3	5	7	9	11	13	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
-14	Arith Modular	0	0	1	2	1	4	0	2	4	6	8	10	12	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-14	Isomorphism	1	2	1	2	1	4	7	2	4	6	8	10	12	14	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-15	Arith Modular	0	1	0	1	0	3	6	1	3	5	7	9	11	13	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
-15	Isomorphism	1	1	3	1	5	3	6	1	3	5	7	9	11	13	15	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
-16	Arith Modular	0	0	2	0	4	2	5	0	2	4	6	8	10	12	14	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
-16	Isomorphism	1	2	2	4	4	2	5	8	2	4	6	8	10	12	14	16	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
-17	Arith Modular	0	1	1	3	3	1	4	7	1	3	5	7	9	11	13	15	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
-17	Isomorphism	1	1	1	3	3	1	4	7	1	3	5	7	9	11	13	15	17	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
-18	Arith Modular	0	0	0	2	2	0	3	6	0	2	4	6	8	10	12	14	16	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
-18	Isomorphism	1	2	3	2	2	6	3	6	9	2	4	6	8	10	12	14	16	18	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
-19	Arith Modular	0	1	2	1	1	5	2	5	8	1	3	5	7	9	11	13	15	17	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
-19	Isomorphism	1	1	2	1	1	5	2	5	8	1	3	5	7	9	11	13	15	17	19	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
-20	Arith Modular	0	0	1	0	0	4	1	4	7	0	2	4	6	8	10	12	14	16	18	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-20	Isomorphism	1	2	1	4	5	4	1	4	7	10	2	4	6	8	10	12	14	16	18	20	1	2	3	4	5	6	7	8	9	10	11	12	13	14

Figure 9. Table of Modular Lattice Grid columns in the 2nd quadrant as modular arithmetic and isomorphism.

3 Hyperbolic Sieve of Primes

The construction of hyperbolas in our Modular Lattice-Grid is done using the generic $H(x, y) = x * y = n^2$ equation.

Figure 10. The Hyperbolic Sieve of Primes and Divisors.

Thus, the line $x = y$ becomes an axis of symmetry and divides the quadrant with the hyperbolas into two halves: (1) the bottom and (2) the top.
Dots that are crossed by hyperbolas are demarcated with the X signal.
In the bottom half, the hyperbolas will pass over the dots that obey $x * y = n^2$. These dots are located on the green $x = k^2 * y$ lines.
In the top half, the hyperbolas will pass over the dots that also obey $x * y = n^2$. These dots are located on the green $y = k^2 * x$ lines.
Thus, using the integers presented by the line $x = y$, the following properties apply:

- Only one hyperbola crosses only a single dot. It's the hyperbola $x * y = 1$, where. $x = y = 1$. All other dots where $x = y \neq 1$ they will be linked by their hyperbolas to at least 2 other dots.
- All Prime numbers are those whose hyperbolas $x * y = n^2$ cross exactly 3 dots. One in the row $x = y$, another has to be located on row $y = 1$ where $x = (Prime)^2$, and the last in column $x = 1$ where $y = (Prime)^2$.
- All Composite numbers are those in which hyperbolas cross at least one dot on each half that is not located on row $y = 1$ or the column $x = 1$. That is, it is the hyperbolas that cross an odd number of times greater than 3.

3.1 Examples:

Hyperbola $x * y = 4$, cross line $x = y$ in a dot$(x, y) = (2,2)$. This dot is labeled as being 2. This hyperbola $x * y = 4$ also crosses the dots $(1,4)$ and $(4,1)$, both labeled as 1. Because hyperbola $x * y = 4$ crosses 3 dots, number 2 is a Prime number.

Hyperbola $x * y = 16$, cross line $x = y$ in a dot $(x, y) = (4,4)$. This dot is labeled as being 4. This hyperbola $x * y = 16$ also crosses the dots $(1,16)$ and $(16,1)$, both labeled as 1. This hyperbola $x * y = 16$ also crosses the dots $(2,8)$ and $(8,2)$, both labeled as 2. Because hyperbola $x * y = 16$ crosses more than 3 dots, number 4 is a Composite number.

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5 References

[1] *The On-Line Encyclopedia of Integer Sequences*, oeis.org, Available online at http://oeis.org.