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# A two-level location allocation and a static dial-a-ride problem

Kirti Sharma<sup>a</sup>, V.P. Singh<sup>a,\*</sup>, Debjani Chakraborty<sup>b</sup>

<sup>a</sup>*Department of Mathematics, Visvesvaraya National Institute of Technology Nagpur, Nagpur - 440010, Maharashtra, India*

<sup>b</sup>*Department of Mathematics, Indian Institute of Technology Kharagpur, Kharagpur-721302, West Bengal, India*

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## Abstract

This paper is concerned with the two-level modelling of the service set-up planning of a logistic service provider company. The problem discussed in this work is a combination of location allocation problem and a static dial-a-ride problem. The objective of the upper level decision maker is to decide the location of customer service centres in a city that maximizes the service coverage area and are located at prime locations in the city. The problem at the lower level comprises of a set of pickup locations and a corresponding set of drop locations for a given set of customers. The objective of the lower level decision maker is to design routes in such a way that satisfies the needs of all the customers in the network (picking them up from pickup locations and dropping them off at their corresponding drop locations) without violating the constraints. The transportation service at second level is carried out by vehicles providing shared service. The problem consists of designing a set of minimum-cost vehicle routes satisfying vehicle routes satisfying capacity, precedence and pairing constraints. In this work, a solution methodology based on the combination of K-means algorithm and nearest neighbour approach is proposed to solve the proposed problem. The problem finds its application in door-to-door transportation for elderly and disabled people. The problem can also be used by private sharing cab provider companies to generate efficient routes for a given set of customers. To present the applicability of the proposed method, computational experiments are performed on randomly generated small to medium sized instances.

*Keywords:*

Transportation and logistics, Network, Clustering algorithms, Dial-a-ride problems, Location allocation problem.

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## 1. Introduction

With the increase of urbanization in developing countries, the need to transport goods and people from one location to another location also increase at a rapid pace. For last mile connectivity, people often depend on their personal mode of transport and this increasing dependency on personal mode of transport results in clogged and congested roads, high level of energy consumption along with the environmental and economic costs, causing noise and air pollution. However, a public mode of transportation,

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\*Corresponding author

*Email addresses:* [unik96kirti@gmail.com](mailto:unik96kirti@gmail.com) (Kirti Sharma), [vpsingh@nth.vnit.ac.in](mailto:vpsingh@nth.vnit.ac.in) (V.P. Singh), [debjani@maths.iitkgp.ac.in](mailto:debjani@maths.iitkgp.ac.in) (Debjani Chakraborty)

either Public Bus transportation or metro transportation resolve the problem of congestion and pollution but the problem of last mile connectivity still remain unresolved. In order to solve the economic and environmental problems caused by private mode of commutation and still enjoy last mile connectivity; a sustainable commuting mode which can be used for transporting transporting customer from door to door is needed. In [1], the economic and environmental benefits of carpooling for potential users have been defined and the behaviour of carpooling for community is analyzed and it is then referred as a sustainable commuting mode. The fuzziness and flexibility incorporated by carpooling is considered in [2] and a new method which merges static network (i.e. public transport) and dynamic/fuzzy network (i.e. carpooling) better while retaining the desired flexibility is presented in [2]. Various modifications of genetic algorithm are used for solving single objective and multi-objective taxi carpooling path optimization models in [3]. With the introduction of carpooling, a need to find optimal routes also arises. In [4], various clustering algorithms are discussed for trip distribution and generation and traffic zone division. A modified cumulative accessibility (MCA) measure is proposed in [5] which incorporate actual travel demand information and is then reflected into the MCA measure by evaluating public transportation and private vehicle transportation (PVT) trip diaries. K-means clustering technique is used for recognizing patterns to characterize the demands of industrial transportation in [6]. The clustering algorithm is modelled as a matching problem between two disjoint sets, namely centroids and data points in [7]. The proposed algorithm is then used to maximize the purity and similarity in each cluster simultaneously.

The transportation system in which a vehicle provides door-to-door transportation service which moves a large number of passengers with personalized service request is known as dial-a-ride problem(DARP). In DARP, a set of customers make a request for service. Each customer wants to be picked up from one given location and to be dropped at a different location. Because of the impressive nature and varied real life applications of the problem, it has attracted various academicians and researchers in recent decades. A literature survey providing the classification of various variants of DARP and its different solution methodologies is given in [8]. DARP with electrical vehicles is considered in [9] in which electric vehicles are recharged by swapping their batteries with a charged battery from any battery swap station. Three enhanced evolutionary variable neighbourhood search algorithms are proposed in [9] for scheduling vehicle routes for customers with special needs. A customized lagrangian relaxation algorithm is proposed in [10] to solve an integer non linear programming model to find optimal routes for automated taxis such that the system profit is maximized. A Bi-objective on-demand bus routing problem is handled by using a large neighbourhood search heuristic method in [11]. The first objective of the problem is to assign a departure and a pickup bus station to every passenger from their set of potential departure and arrival bus stations. The second objective of the problem is to schedule the routes so that each request is fulfilled in time with an objective of minimizing total travel time. A two-phase branch and cut and price algorithm is proposed in [12] to solve the realistic dial-a-ride problem keeping capacity constraint, maximum riding time constraint, man power planning constraints and time window constraints. The lunch

break constraint of staff members is generalized and the problem is solved with an objective of satisfying maximum number of requests while traversing least travel distance. A hybrid algorithm to solve single and multiple depot DARP variants where both demand and vehicle fleet are heterogeneous is proposed in [13]. Different anticipatory algorithms are investigated for solving various dynamic and stochastic transportation problems in [14]. The real world data is synchronized and besides considering new events, stochastic information about future events is also incorporated. A variant of mixed fleet heterogeneous dial-a-ride problem is proposed for a container truck-routing problem in [15]. Route planning for the combination of dynamic and static transportation requests is attempted by using heterogeneous fleet of vehicles in [16]. A combination of variable neighbourhood search and insertion heuristic is used to solve static and dynamic requests respectively. A mixed-integer optimization model is presented in [17] for solving a heterogeneous dial-a-ride problem where en-route modifications of vehicle inner configuration are needed. A meta-heuristic algorithm based on large neighbourhood search is proposed to solve the problem.

In order to provide these dial-a-ride services some customer service centres should be established in the city which can be easily accessed by the customers. In the terminology of operations research, this problem is known as location allocation problem. The problem finds its application in various real-life situations such as resource management at the time of a disaster [18], setting up medical evacuation facilities [19], site selection for infectious waste disposal in a city [20], site selection for construction of hospital [21] and many more. A location allocation problem for determining the position of relief distribution centres that maximizes the affected coverage area and minimizes the cost incurred and time spent in relief operation is modelled and solved in [18]. An  $\epsilon$ -constraint method is proposed for solving medical evacuation problem in [19] with the objective of maximizing the expected demand coverage area, minimizing the maximum number of mobile aeromedical staging facilities (MASFs) and minimizing the total number of MASFs relocations in deployment phase. An integrated method based on the combination of Fuzzy AHP and Fuzzy TOPSIS method is proposed to solve the facility location allocation problem for infectious waste disposal site in a city in [20]. An improvised version of genetic algorithm is used for solving hospital site selection problem in [21]. Two new solution methods namely constraint programming method and a hybrid of K-means and genetic algorithm are developed for generating exact and approximate solution for solving p-hub location allocation problem in [22].

In this paper, a mathematical model and a solution algorithm are proposed to address the problem of serving customers and define vehicle routes on an urban road network. The model considers vehicle owner's cost by incorporating travel cost while the design of routes of vehicles is related to a dial-a-ride problem (DARP) in operations research. The model also allows ride-sharing, in order to increase transportation efficiency and reduce pollution and congestion. A hybrid K-means and nearest neighbour algorithm is developed to solve the proposed NP-hard routing problem for real case-study applications.

The main contributions of this paper are : firstly, we formulated a mathematical model to solve a two-level location allocation dial-a-ride problem, allowed for ride sharing between passengers. Secondly, we developed a hybrid algorithm based on the combination of K-means and nearest neighbour algorithm which is able to approach the near optimal solution to the proposed model. Thirdly, the application of the discussed problem statement and its solution methodology reveals the potential importance of cab sharing facilities and their impact on traffic congestion, pollution and customer satisfaction.

This paper is structured as follows: Firstly, the literature done regarding location allocation and DARP is reviewed in Section 1. K-means clustering algorithm and nearest neighbour algorithm are discussed in Section 2. A mathematical model for two-level location allocation and dial-a-ride problem is introduced in Section 3. Section 4 represents the solution methodology based on K-means and nearest neighbour algorithm to solve the problem. A small illustrative numerical example is presented in Section 5. The paper ends with conclusion in section 6.

## 2. Preliminaries

### 2.1. K Means clustering algorithm

K-means clustering is one of the simplest and most popular unsupervised machine learning algorithm that group similar data points together. In this method, the data points are assigned to clusters in such a way that the sum of squared distance between data points and centroids is as small as possible. Here, K determines the number of pre-defined clusters that needs to be created in the process. The working of K-means clustering algorithm can be summarized as follows:

Step 1: Randomly initialize K centroids.

Step 2: Compute the distance of all data points from each of K cluster centroids.

Step 3: Assign each data point to its closest centroid.

Step 4: Compute the new centroid of each cluster.

Step 5: Repeat Step 2, 3 and 4 until cluster centroids do not change.

### 2.2. The Nearest Neighbour algorithm

Nearest neighbour algorithm is one of the first algorithms based on the greedy approach to solve the travelling salesman problem approximately. As the name suggests, in this algorithm, the salesman starts from a random city and repeatedly visits the nearest city until all cities have been visited. The working of nearest neighbour algorithm in the case of vehicle routing problem can be summarized as follows:

Step 1: Start the journey from depot node, O.

Step 2: Set  $i = O$  and  $V = \{1, 2, \dots, n\}$ ; where  $V$  is the set of points that is to be visited.

Step 3: Let  $j \in V$  such that  $c_{ij} = \min\{c_{ik} : k \in V\}$ .

Step 4: Connect  $i$  to  $j$  and set  $V = V \setminus \{j\}$  and  $i = j$ .

Step 5: Repeat Step 3 and 4 until  $V = \emptyset$ .

Step 6: Connect  $i$  to depot node  $O$  to form a Hamiltonian cycle.

### 3. A two-level location allocation and a static dial-a-ride problem

The dynamic version of location allocation problem is to locate a set of new facilities such that the operational cost of serving the customers in the network is minimized and an optimal number of facilities have to be placed in an area of interest in order to satisfy customer demands. The static version of a dial-a-ride problem consists of serving a set of customers by picking them up from a set of pickup locations and dropping them at a set of their corresponding drop locations. The current version of the problem deals with a combination of location allocation problem and a dial-a-ride problem. This work represents the bi-level nature of the routing problem. In the problem discussed in this work, the upper level decisions are mainly controlled by logistic service provider companies and their objective is to construct customer service centres at a pre-specified number of locations in a city. The construction of these customer service centres divide the city into a pre-specified number of zones and thus the service coverage area offered by the company increases in the city. The lower level decision makers are the vehicle owners of each zone who decide the sequence in which the customers are served. In this problem, there are two objectives, which are to be dealt at two different levels:

**Objective 1:** Divide the city into a pre-defined number of zones.

**Objective 2:** Considering each zone center as a depot and assuming that each depot has ample amount of fleet to cater the needs of the customers belonging to that zone, design a set of routes in which every customer is picked up from their pickup location and dropped at their respective drop location in the minimum cost.

In this work, the nature of the network has been assumed to be known precisely and well in advance. The pickup locations and drop locations of all the customers are also known well in advance.

#### 3.1. Assumptions of the model

Before discussing the mathematical model of a two-level location allocation and a static dial-a-ride problem, we present some basic assumptions regarding the model that will be used throughout the paper. The very first assumption is regarding the network and it states that the network under consideration is symmetric and follows triangle inequality, i.e. if the cost of traversal from node  $i$  to node  $j$  is given by  $c_{ij}$ , then  $c_{ij} = c_{ji}$  and  $c_{ij} \leq c_{ik} + c_{kj}$ . In this work, we have assumed that the edge weights represent

the cost of traversal of edges and is given in deterministic form. A set of assumptions is also made for the pickup locations and drop locations of every customer present in the network. It is assumed that the pickup locations and drop locations of every customer are known exactly and well in advance. The pickup location and drop location of any customer is visited exactly once. It is also assumed that no two customers have the same pickup location or same drop location. It is also assumed that the drop location of no customer is same as the pickup location of any other customer or vice versa. A set of assumptions is also made for the journey of fleet of vehicles in the network. The journey of every vehicle is assumed to originate and terminate at their corresponding zone center only. It is also assumed that the pickup location and drop location of a customer belong to the same zone. It is assumed that the pickup location and drop location of a vehicle is traversed by a single vehicle only. It is also assumed that the fleet of vehicles present at depot is homogeneous; i.e. all the vehicles have the identical operating costs and they all have the same carrying capacity.

### *3.2. Applications of the model*

The real world application of a two-level location allocation and a static dial-a-ride problem include among others the planning and execution of a logistic service company in a new city. When a logistic service company plans to set their foot in a new city, planning takes place at two levels. The planning at first level includes setting up a pre-decided number of consumer service centres, from where the consumers will avail the services. The location for these consumer service centres is to be chosen in a way that maximizes the service coverage area offered by the company and these service centres partition the city. At the next level, i.e., at the execution level, the service demands of the customers arrives in the form of tuples, where first element of tuple represents the pickup location and the second element represent the drop location. In a profit centric world, the problem statement reveals the working of a logistic service provider company, whereas, in a service centric world, the model can be applied in situations where elderly or handicapped people constitute the set of customers who want to visit their desired hospitals (corresponding drop locations) for their regular check-up. In this scenario, the location of customers serves as the pickup location. The solution of the modelled problem and its application in a profit centric world helps logistic service provider companies to plan their businesses and increase their profit significantly whereas the solution of the modelled problem and its application in a service centric world will help in making a better environment for specially abled people and thus bring equity in the society.

### *3.3. Mathematical model*

A two-level location allocation and a static dial-a-ride problem is represented by a complete weighted graph  $G = (V, E)$  where  $V$  is the set of vertices and  $E$  is the set of edges joining these vertices. The set of vertices includes pickup locations and drop locations of the customers. In the first stage of a two level location allocation and a static dial-a-ride problem, the network is divided into a pre-specified number ( $K$ ) of zones and the center points of these zones  $c_i; i = 1, 2, \dots, K$  are defined. The motive behind

dividing the network into zones is to increase the service coverage area and thus the service satisfaction. The center points of these zones, (hereafter referred as zone centres) acts as depot from where all the vehicles serving that zone must start and finish their journeys. It is assumed that the pickup location and drop location of a customer belong to the same zone. The second stage of problem solving corresponds to finding a sequence of routes that should be traversed in a way such that each customer of every zone is served by picking them up from their pickup locations and dropping them at their corresponding drop locations. While solving the routing problem at the second stage, the capacity constraints of the vehicle, pairing constraints and precedence relationship between pick up point and drop points is always kept in mind. The capacity constraint of the problem states that at any moment, vehicle is not supposed to carry more than  $Q$  customers, where  $Q$  is the carrying capacity of the vehicle. According to the precedence relationship, no drop location of a customer is traversed before traversing their pickup point. The coupling (pairing) relationship states that the pickup location and drop location of every customer is served by the same vehicle only.

A mathematical model representing a two-level location allocation and a static dial-a-ride problem is given as follows:

$$\text{Minimize} \quad \sum_{k=1}^K \sum_{i=1}^{|P_k|} \text{dist}_{ic_k} y_{ik} \quad (1)$$

$$\text{Minimize} \quad \sum_{k=1}^K \sum_{j \in S_k} \sum_{i \in S_k} \sum_{v_k \in V_k} c_{ij} x_{ijv_k} \quad (2)$$

$$\sum_{k=1}^K y_{ik} = 1 \quad \forall i \in P \cup D \quad (3)$$

$$\sum_{i \in P \cup D} y_{ik} = \left\lceil \frac{n}{K} \right\rceil \quad \forall k \in \{1, 2, \dots, K\} \quad (4)$$

$$\sum_{i=1}^n y_{ik} - \sum_{i=1}^n y_{n+ik} = 0 \quad \forall k \in \{1, 2, \dots, K\} \quad (5)$$

$$\sum_{k=1}^K \sum_{j \in S_k} \sum_{i \in S_k} \sum_{v_k \in V_k} x_{ijv_k} = 1 \quad \forall i \in S_k, v_k \in V_k, k = \{1, 2, \dots, K\} \quad (6)$$

$$\sum_{j \in S_k - \{c_k\}} x_{ijv_k} - \sum_{j \in S_k - \{c_k\}} x_{jiv_k} = 0 \quad \forall i \in S_k - \{c_k\}, v_k \in V_k, k = \{1, 2, \dots, K\} \quad (7)$$

$$\sum_{i \in S_k} \sum_{j \in P_k} x_{ijv_k} - \sum_{i \in S_k} \sum_{j \in D_k} x_{ijv_k} \leq Q \quad \forall k = \{1, 2, \dots, K\} \quad (8)$$

$$\sum_{i \in S_k} \sum_{j \in P_k} (c_{ij} - c_{in+j}) x_{ijv_k} \leq 0 \quad (9)$$

$$\sum_{j=1}^n x_{ijv_k} - \sum_{j=1}^n x_{in+jv_k} = 0 \quad \forall i \in S_k - \{c_k\} \quad (10)$$

$$\sum_{j=1}^{2n} \sum_{c \in C} x_{cjv_k} = K \quad \forall j \in S_k, v_k \in V_k, k = \{1, 2, \dots, K\} \quad (11)$$

$$y_{ik} = \{0, 1\} \quad i = \{1, 2, \dots, 2n\}; k = \{1, 2, \dots, K\} \quad (12)$$

$$x_{ijv_k} = \{0, 1\} \quad i, j = \{1, 2, \dots, 2n\}; k = \{1, 2, \dots, K\} \quad (13)$$

In the mathematical model represented above, the objective represented by eq. (1) ensures that the distance between intra-cluster points is minimum and this represents the objective of location allocation problem whereas the objective function represented by eq. (2) corresponds to finding a minimum distance tour between all the nodes of zone  $k$ . The constraint represented by eq. (3) ensures that a point belongs to exactly one cluster and the constraint represented by eq.(4) ensures that each point is assigned to one or the other cluster. The constraint represented by eq.(5) ensures that if pickup point of a customer (denoted by  $i$ ) belongs to a zone  $k$  then their drop point (denoted by  $n + i$ ) also belong to cluster  $k$ . The constraint represented by eq. (6) ensures that each point, whether it is a pickup point or a drop point is traversed exactly once. The constraint represented by eq. (7) ensures that the route of a vehicle originates and terminates at zone centres only. Eq. (8) represents the capacity constraint; i.e. at no time the load in the vehicle exceeds its carrying capacity and eq. (9) represents the precedence relationship, i.e., the drop point of a customer is traversed only after traversing its pickup point. The constraint given by eq.(10) represents the coupling relationship i.e. any vehicle traversing the pickup point of a customer must also traverse their drop point and vice versa. The constraint represented by eq. (11) ensures that exactly  $K$  vehicles start their trip from  $K$  zone centres at a point of time. The variable  $y_{ik}$  represented in eq. (12) is a binary variable which takes the value 1 when the pickup/drop location  $i$  belongs to  $k^{th}$  cluster and 0 otherwise. The binary variable  $x_{ijv_k}$ , represented by eq. (13) is again a binary variable which takes the value 1 when the arc  $ij$  is traversed by a vehicle  $v$  of zone centred at  $c_k$  and 0 otherwise.

#### 4. Solution Methodology

The solution methodology to solve a two-level location allocation and a static dial-a-ride problem has been divided into two stages. In the first stage, the city is partitioned into a pre-defined number of zones. In this work, the partitioning of city is done by using K-means clustering algorithm. Proper care has been taken to partition the city into equi-sized clusters. This step includes clustering the customers as well as finding the centroids of those clusters. This part of the methodology is represented by Algorithm 1. In the second stage of problem solving, minimum weighted travelling salesman routes originating and terminating at cluster centroids serving all the customers exactly once are to be determined while keeping

various constraints such as pairing constraint, precedence relationship and capacity constraints in mind. This part of solution methodology is given by Algorithm 3.

#### 4.1. Flowchart of the proposed model

The flowchart of the proposed method is given by Figure 1 and Figure 2. The total process of problem solving is represented by using two flowcharts.

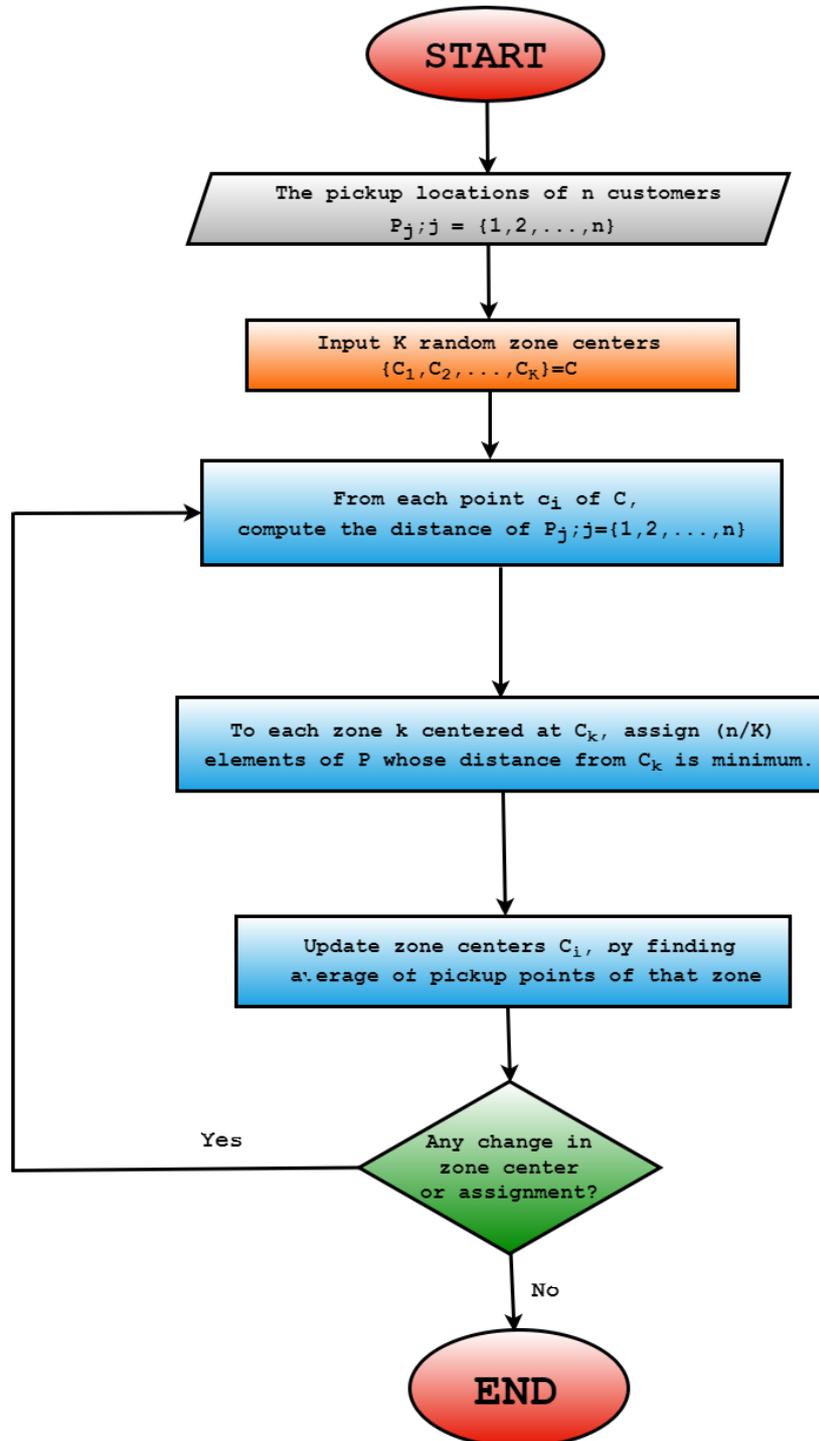


Figure 1: Flowchart for stage 1

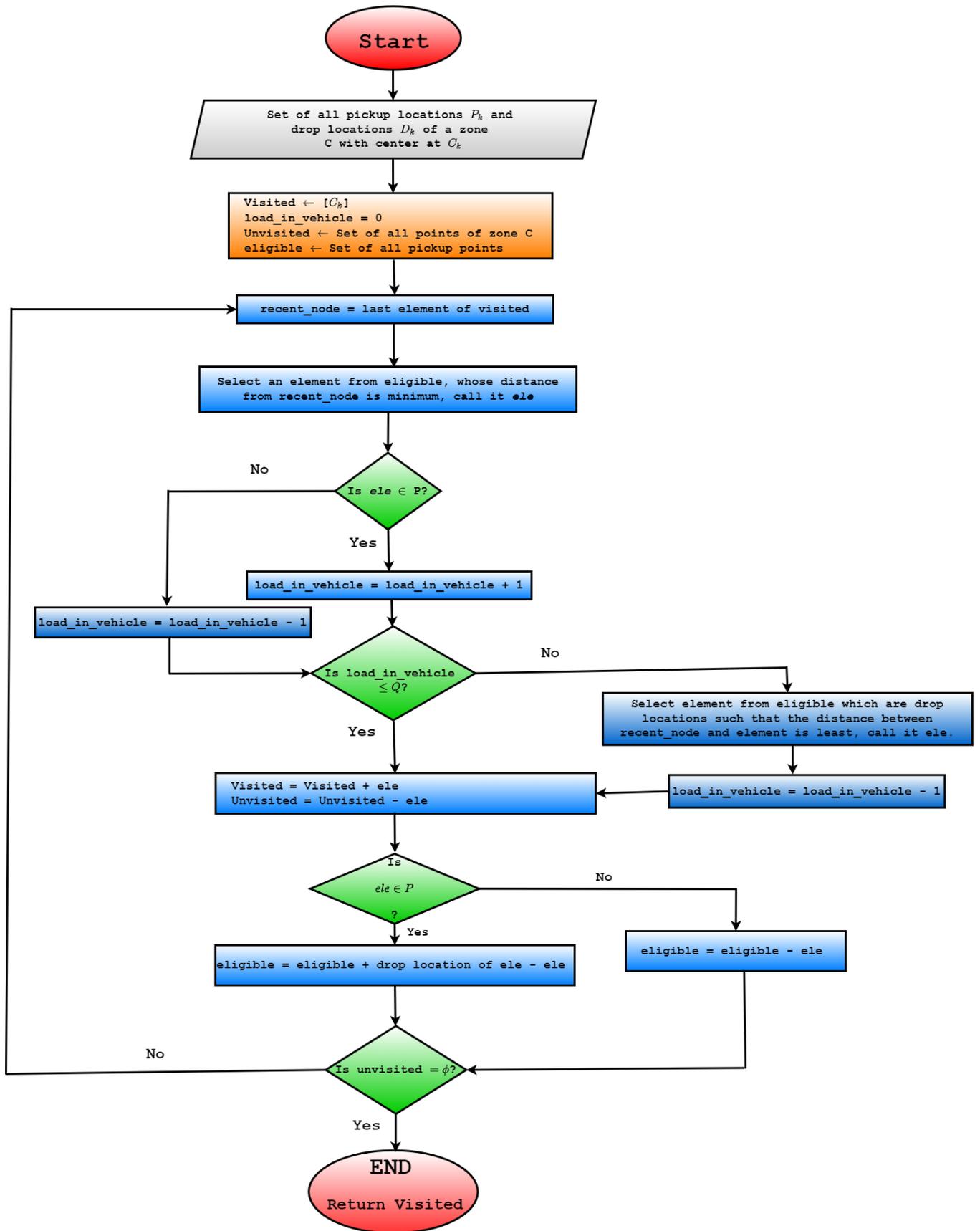


Figure 2: Flowchart for stage 2

#### 4.2. Algorithm

The algorithm of the proposed model is divided into two major parts. In the first part, the clustering of customers into equi-sized clusters is done. This part of methodology is presented by using Algorithm 1. It returns the  $K$  zone centres and set of customers belonging to each zone centre. In the second part, the clusters of the customers formed in step 1 are served and a least cost route corresponding to each cluster is obtained. This part of the methodology is shown by Algorithm 3. Algorithm 2 returns the set of customers that are eligible to be visited on the basis of nodes that have been traversed. Table ?? comprises the symbols and their descriptions as used in various algorithms.

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**Algorithm 1** An algorithm to divide  $n$  customers into  $K$  equi-sized clusters.

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**Input:** Set of data points,  $P$ , representing the pick up locations of  $n$  customers, Number of clusters  $K$  and  $\epsilon$ .

**Output:** Centres  $\{c_1, c_2, \dots, c_k\}$  implicitly dividing  $P$  into  $K$  equi-sized clusters and  $S_i, i = 1, 2, \dots, K$  representing the set of customers in each cluster.

```

1:  $t \leftarrow 0$ 
2: Randomly initialize  $K$  centroids;  $c_1^t, c_2^t, \dots, c_K^t$ 
3: repeat
4:    $t \leftarrow t + 1$ 
5:    $S_j \leftarrow \phi$  for all  $j = 1, 2, \dots, K$ 
6:    $j \leftarrow 1$ 
7:   repeat
8:     repeat
9:       for each  $x_p \in P$  do:
10:         $j = \operatorname{argmin}_i |x_p - c_i^t|^2$ 
11:         $S_j \leftarrow S_j \cup \{x_p\}$ 
12:         $P \leftarrow P - \{x_p\}$ 
13:      until  $|S_j| \leq \frac{n}{K}$ 
14:       $j \leftarrow j + 1$ 
15:    until  $j \leq K$ 
16:  end for
17:  for each  $i = 1$  to  $K$  do:
18:     $c_i^{t+1} \leftarrow \frac{1}{|c_i^t|} \sum_{x_j \in S_i} |x_j|$ 
19:  end for
20: until  $|c_i^{t+1} - c_i^t| \leq \epsilon$ 
21: return  $S_j$  and  $C_j$  for  $j = 1, 2, \dots, K$ . .

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**Algorithm 2** An algorithm that returns the eligible set of data points.

---

**Input:** Set of pickup points,  $P$ , and set of drop points,  $D$ , where

$$P = \{p_1, p_2, \dots, p_n\}.$$

$$D = \{d_1, d_2, \dots, d_n\}.$$

**Output:** Set of eligible data points.

```
1: eligible  $\leftarrow P$ 
2: visited  $\leftarrow$  the set of nodes that have been visited
3: for ele  $\in$  visited do:
4:   if ele  $\in P$ , i.e., ele =  $P_i$  for some  $i = 1, 2, \dots, n$ 
5:     eligible = eligible  $\cup D_i$ 
6:   end if
7: end for
8: for ele  $\in$  eligible do:
9:   if ele  $\in$  visited:
10:    eligible = eligible  $- ele$ 
11:   end if
12: end for
13: return eligible.
```

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**Algorithm 3** An algorithm that returns the sequence in which the customers of each cluster are to be served.

---

**Input:** The pickup point coordinates  $P_k$  and the drop coordinates  $D_k$  of all customers belonging to a cluster  $k$  with center at  $c_k$ .

**Output:** The nearest neighbour route serving each customer of that cluster.

```

1: Compute a distance matrix that comprises of the distance between all points  $V$  of a cluster  $k$ , where
    $V = P_k \cup D_k \cup c_k$ 
2:  $visited \leftarrow [c_k]$ 
3:  $eligible \leftarrow P_k$ 
4:  $unvisited \leftarrow P_k \cup D_k$ 
5: while  $unvisited \neq \phi$  do :
6:    $recent - node \leftarrow$  last element of visited
7:   for  $ele \in eligible(visited)$  do:
8:      $element = \operatorname{argmin}_{ele} \{cost[recent - node][ele] : ele \in eligible(visited)\}$ 
9:   end for
10:  if  $element \in P_k$  do:
11:    load in vehicle  $\leftarrow$  load in vehicle + 1
12:  end if
13:  if load in vehicle  $\leq Q$  do:
14:     $visited \leftarrow visited \cup ele$ 
15:     $unvisited \leftarrow unvisited - ele$ 
16:     $eligible \leftarrow eligible \cup$  Drop location of  $ele - ele$ 
17:  else:
18:    for  $ele \in eligible \cap D_k$  do :
19:       $element = \operatorname{argmin}_{ele} \{cost[recent - node][ele] : ele \in eligible(visited) \cap D_k\}$ 
20:    end for
21:     $visited \leftarrow visited \cup ele$ 
22:    load in vehicle  $\leftarrow$  load in vehicle - 1
23:     $unvisited \leftarrow unvisited - ele$ 
24:     $eligible \leftarrow eligible - ele$ 
25:  end if
26: end while
27: return  $visited$ 

```

---

## 5. Experimental Results

Let us consider a small instance with 16 customers in a network. Suppose that the customers in the network have pre-defined pickup locations and drop locations. It is assumed that the cost matrix stores the information about distance between various nodes (pickup locations and drop locations) in the network. The capacity of the fleet of vehicles is assumed to be of 4 people. For this example, the distance between various nodes is calculated by using Euclidean distance. Figure 3 represents the pick-up and drop locations of all 16 customers in the network.

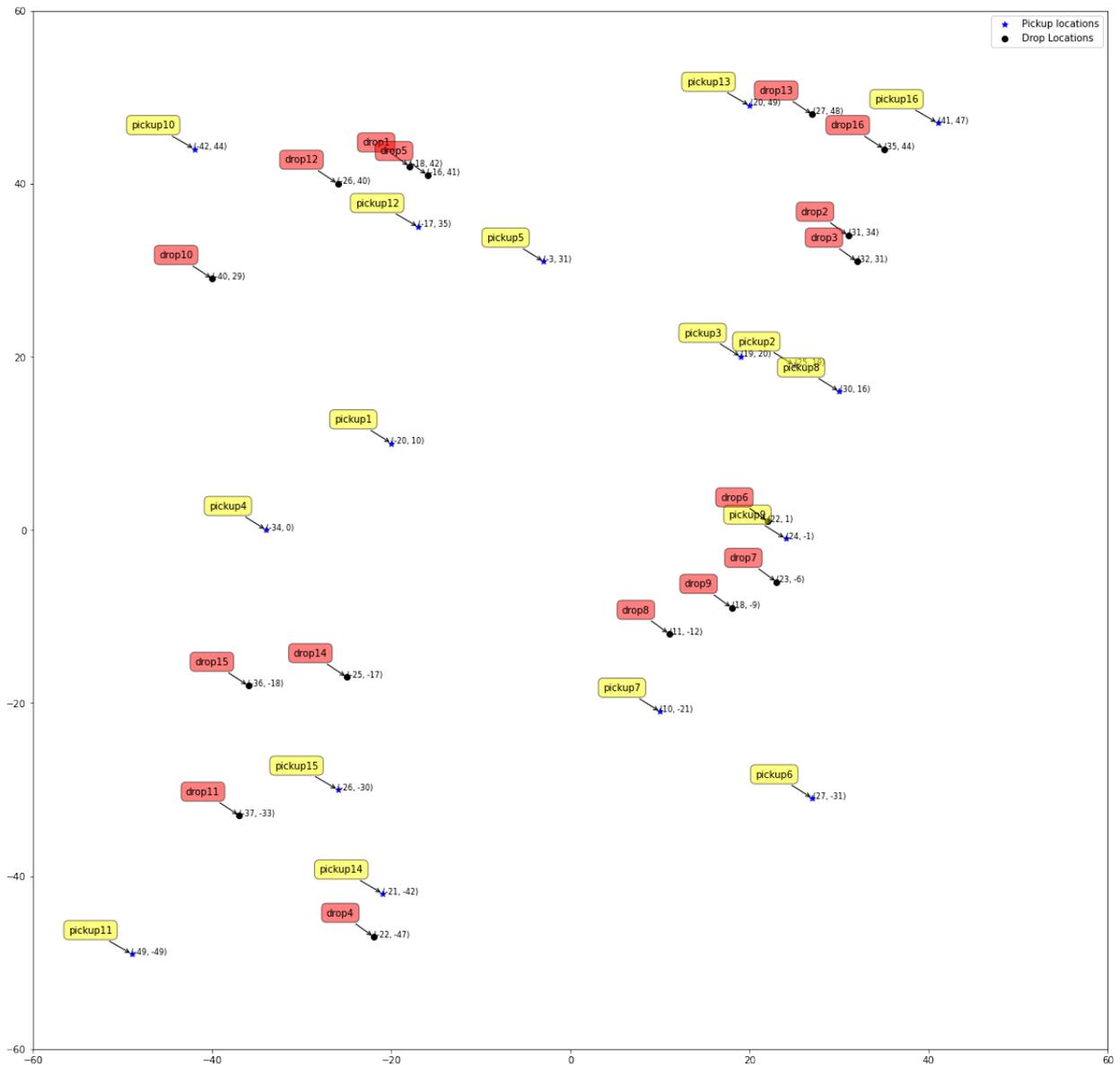


Figure 3: Pick-up and Drop locations of the customers

The solution finding procedure can be divided into two stages where stage 1 deals with clustering of customers into a pre-defined (here,  $K = 4$ ) number of clusters and finding the centroids of those clusters.

Figure 4 corresponds to this stage of solution finding .

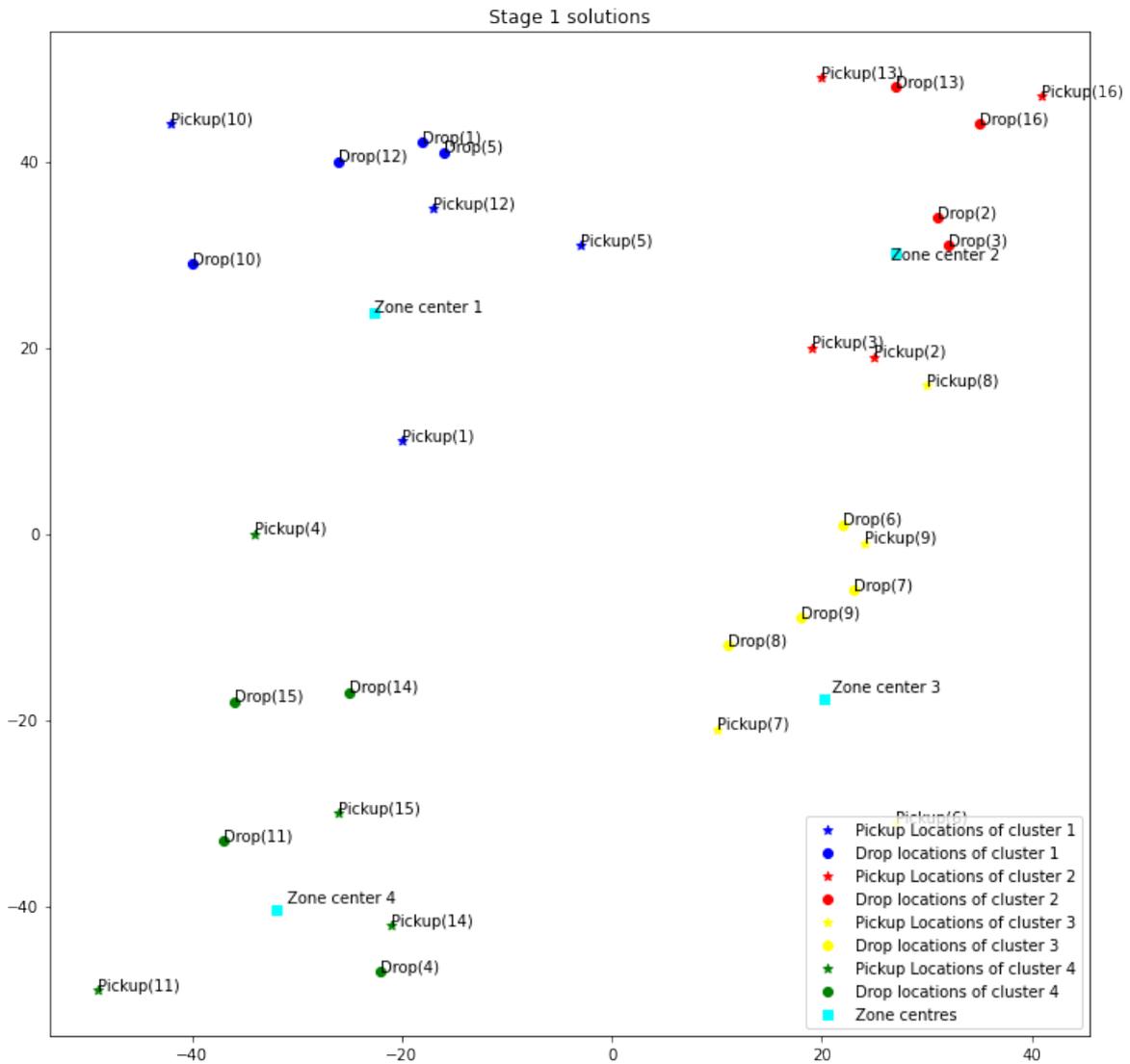


Figure 4: Clustering of customers into equi-sized clusters

The stage 2 of solution finding corresponds to finding a sequence of nodes (corresponding to each cluster) in which the vehicles should operate in each cluster so that the operational cost is minimum. Figure 5 corresponds to the nearest neighbour route corresponding to each cluster formed in stage 1 (as given in Figure 4).

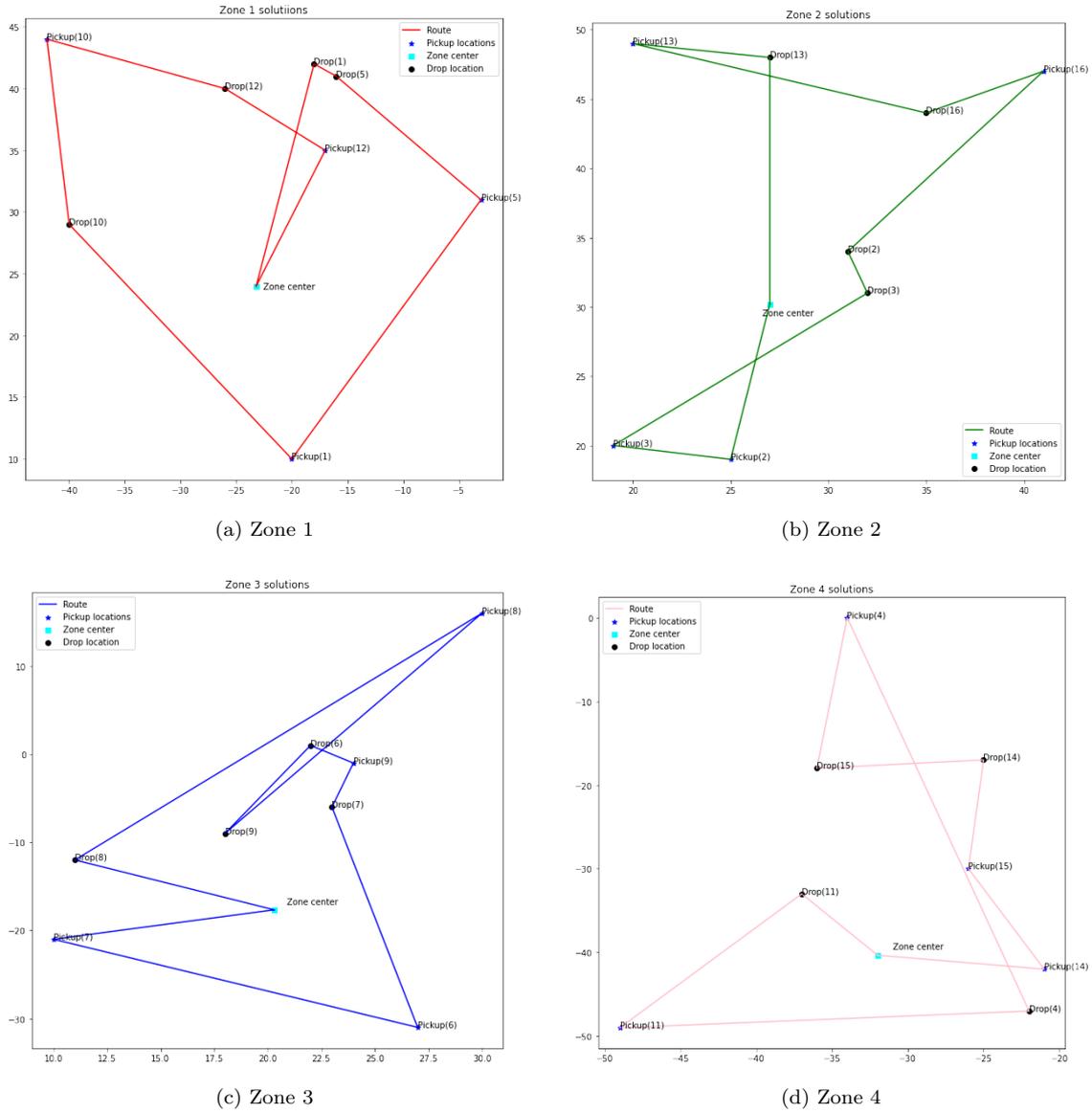


Figure 5: Nearest Neighbour routes in all zones (clusters)

## 6. Conclusion

In this paper, a mathematical model of two-level location allocation and a static dial-a-ride problem has been presented. An integrated algorithm composed of K-means algorithm and nearest neighbour algorithm to solve such a problem has also been presented. The problem is modelled as a bi-level optimization problem where objective at leader level is to find locations of customer service centres in a city and assigning customers to these service centres on the basis of requests made by them. The objectives at lower level is to decide the sequence in which the customers are to be served. The customer service requests in this work are of pickup-drop types; i.e., corresponding to each customer there exists a pickup location (the location from where the customer has to be picked up) and a drop location (the location

where the customer has to be dropped).

The approach used in this work is a combination of K-means algorithm and nearest neighbour algorithm. In the first stage of problem solving, the K-means algorithm is used to segregate the customers into clusters and to locate customer service centres. These customer service centres are then used as zone depots. In the second stage of problem solving, nearest neighbour algorithm is used to find a sequence of routes in which the customers are to be served. A numerical example of small size is also solved using the proposed approach. The proposed two-level problem can further be used by emergency service providers to first locate the facility centres and then to formulate the routes in which customers are to be served. The model can further be used by logistic service providers to plan the extension of their services in a new city. The imprecise and uncertain nature of road network can also be taken into account while solving the problem modelled in this work.

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