# Filter Characteristic of a Triangular Periodic Transmission Line in Micro-Strip Structure 

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#### Abstract

- we have studied the wave propagation through a microstrip transmission line which is a periodic structure and the width of which is modulated by a triangular shape. Various characteristics like propagation constant, the characteristic impedance of the microstrip transmission have been studied. Here the periodic structure is divided into unit cells and the electromagnetic field supported by each unit cell has been determined analytically. The wave amplitude transmission (WAT) matrix of each radial section of a cell is determined and then combined to determine the overall WAT matrix of the unit cell. With the help of the WAT matrix, various characteristics of the structure have been explored. The expression thus obtained is simulated using MATLAB software. The nature of the wave transmission through the microstrip line has been studied for different dimensions of the line and different ranges of frequencies. It is seen that this periodic structure exhibits alternate pass bands and stop bands. For a particular dimension, the microstrip transmission line has been fabricated. The wave transmission through the line and the presence of pass band and stop band has been tested experimentally.


Keywords—Microstrip transmission lines, periodic structures, microwave filter

## I. Introduction

Microstrip transmission lines are the subject of much research activities among scientists due to their unique advantages, especially their low weight and low cost of fabrication. Moreover with the help of periodic structures, as in our case, it is possible to realize a microwave filter as these structures exhibit alternate pass-band and stop-bands. This type of periodic structure can be realized in various forms like sinusoidal, triangular. We have studied the wave propagation through a microstrip transmission line which is a periodic structure and the width of which is modulated by a triangular shape. Various characteristics like propagation constant, the characteristic impedance of the microstrip transmission line have been studied. Here the periodic structure is divided into unit cells and the electromagnetic field supported by each unit cell has been determined analytically. The Wave Amplitude Transmission (WAT) matrix of each radial section of a cell is determined and then combined to determine the overall WAT matrix of the unit cell. With the help of the WAT matrix, various characteristics of the structure have been explored. The expressions thus obtained are simulated using mat lab software. The nature of Periodic transmission through the microstrip line has been studied for different dimensions of the line and different ranges of frequencies. It is seen that this periodic structure exhibits alternate pass-bands and stop-
bands. For a particular dimension, the microstrip transmission line has been fabricated. The wave transmission through the line and the presence of pass-band and stop-band has been tested experimentally.

## II. Literature review

Microstrip transmission lines are the subject of much research activities among scientists due to their unique advantages, especially their low weight and low cost of fabrication. Moreover, with the help of periodic structures as in our case, it is possible to realize a microwave filter as these structures exhibit alternate passbands and stopbands. This type of periodic structure can be realized in various forms. After going through the recent researches over periodic structures we came across various approaches. The wave propagation along a microstrip line with sinusoidal varying width has been studied [1]. This analysis is done using the circuit theory approach and filter-like propagation characteristics of this structure are used for the realization of the microwave filter. The various characteristics of the annular ring microstrip antenna have been obtained using the circuit theory approach [2]. Here the variation in the values of the characteristics has also been shown as a function of the various dimensions of the microstrip antenna. The wave propagation along a microstrip line with triangularly varying width has been studied [3]. We elaborately this concept and include some changes. We also design this microstrip line and verify our idea both theoretically and experimentally.

## III. Mathematical Model Triangular Periodic Transmission Line

The width- modulated periodic structure is depicted in the figure. The periodicity of the structure is $p$, and the minimum and maximum widths are $a$ and $b$ respectively.


Fig 1: Triangular width modulated periodic structure: $\mathrm{p}=$ periodicity, $\mathrm{a}=$ minimum width, $\mathrm{b}=$ =maximum width

A unit cell of the structure is shown in the figure. This cell can be modeled as a combination of two annular sectors joined face to face. Let the outer and the inner radii of the sectors be $r$ and $R$, respectively. The angle of the sector can be defined as

$$
\begin{array}{r}
\frac{b}{2}-\frac{a}{2} \\
\frac{p}{2} \\
\tan \frac{\theta}{2}=\frac{\tan \theta}{2} \\
\theta=2 \arctan \frac{b-a}{p} \ldots \ldots \ldots \ldots  \tag{2}\\
2(R-r)=p \ldots \ldots \ldots
\end{array}
$$

Considering the area of the shaded portion or unit cell,

$$
\begin{aligned}
\frac{\theta R^{2}}{2}-\frac{\theta r^{2}}{2} & =\frac{\theta}{2}\left(R^{2}-r^{2}\right) \\
& =\frac{\theta}{2}(R+r)(R-r)
\end{aligned}
$$

Substituting the value from equation (2), we get,

$$
\begin{array}{r}
\frac{\theta R^{2}}{2}-\frac{\theta r^{2}}{2}=\frac{p}{4} \cdot \theta(R+r) \ldots \ldots \ldots \\
b=\theta R ; a=\theta r ; \\
a=b=\theta(R+r) \ldots \ldots \ldots \ldots \tag{4}
\end{array}
$$

Substituting equation (4) in equation (3), we get,

$$
\begin{equation*}
\frac{\theta R^{2}}{2}-\frac{\theta r^{2}}{2}=\frac{p}{4}(a+b) \tag{5}
\end{equation*}
$$

Substituting equation (1) in equation (5), we get,

$$
\begin{array}{r}
2 \arctan \frac{b-a}{p}\left[\frac{(R+r)(R-r)}{2}\right]=\frac{p}{4}(a+b) \\
2(R+r)(R-r)=\frac{p(a+b)}{2 \arctan \frac{\mathrm{~b}-\mathrm{a}}{\mathrm{p}}} \cdots \cdots \cdots \cdots \cdots \cdots . \tag{6}
\end{array}
$$

Substituting equation (2) in equation (6), we get,

$$
\begin{equation*}
p(R+r)=\frac{p(a+b)}{2 \arctan \frac{\mathrm{~b}-\mathrm{a}}{\mathrm{p}}} \tag{7}
\end{equation*}
$$

From equation (2), we have,

$$
\begin{array}{r}
2(R-r)=p \\
2 R-2 r=p \\
2 R=p+2 r \\
R=\frac{p}{2}+r \ldots \ldots \ldots \ldots . \tag{8a}
\end{array}
$$

And,

$$
\begin{array}{r}
2(R-r)=p \\
2 R-2 r=p \\
2 r=2 R-p \\
r=R-\frac{p}{2} \ldots \ldots \ldots \ldots . \tag{8b}
\end{array}
$$

Substituting the value of equation (8a) in equation (7), we get,

$$
\begin{gathered}
\frac{p}{2}+r+r=\frac{a+b}{2 \arctan b-a / p} \\
2 r=\frac{a+b}{2 \arctan b-a / p}-\frac{p}{4} \\
r=\frac{a+b}{4 \arctan b-a / p}-\frac{p}{4}
\end{gathered}
$$

Substituting the value of equation (8b) in equation (7), we get,

$$
R+R-\frac{p}{2}=\frac{a+b}{2 \arctan b-a / p}
$$

$$
\begin{aligned}
2 R & =\frac{a+b}{2 \arctan b-a / p}+\frac{p}{2} \\
R & =\frac{a+b}{4 \arctan b-a / p}+\frac{p}{4}
\end{aligned}
$$

After obtaining the inner radii and outer radii of the equivalent sector as above, the electromagnetic field supported by the structure may be determined from the following wave equation for z component of the electric field $E_{z}$.

$$
\nabla^{2} E_{z}+k_{1}{ }^{2} E_{z}=0
$$

Where, $\quad k_{1}{ }^{2}=\omega^{2} \mu_{o} \epsilon_{o} \epsilon_{r}$
Where, $\epsilon_{r}$ is the dielectric constant of the substrate on which periodically loaded structure is etched.
$E_{z}$ is independent of $z$ since $h$, the thickness of the structure is assumed to be small compared to $\lambda$, the wavelength of interest.

$$
\begin{gathered}
\nabla^{2} E_{z}+k_{1}{ }^{2} E_{z}=0 \\
\frac{1}{r} \cdot \frac{\partial}{\partial r}\left(r \cdot \frac{\partial E_{z}}{\partial r}\right)+\frac{1}{r^{2}} \cdot \frac{\partial^{2} E_{z}}{\partial \varphi^{2}}+k_{1}{ }^{2} E_{z}=0
\end{gathered}
$$

Assuming $E_{z}$ as a function of r and $\varphi$,

$$
\begin{gathered}
E_{z}=R(r) \emptyset(\varphi) \\
\frac{\partial E_{z}}{\partial r}=\frac{\partial R}{\partial r} \emptyset \\
\frac{\partial^{2} E_{z}}{\partial \varphi^{2}}=\frac{\partial^{2} \emptyset}{\partial \varphi} \cdot R \\
\frac{1}{r} \cdot \frac{\partial}{\partial r}\left(r \cdot \frac{\partial R}{\partial r} \cdot \emptyset\right)+\frac{1}{r^{2}} \cdot \frac{\partial^{2} \emptyset}{\partial \varphi^{2}} \cdot R+k_{1}^{2} E_{z}=0
\end{gathered}
$$

Dividing both sides by $R \emptyset / r^{2}$, we get,

$$
\begin{gathered}
\frac{r}{R} \cdot \frac{\partial}{\partial r}\left(r \cdot \frac{\partial R}{\partial r}\right)+\frac{1}{\varphi} \cdot \frac{\partial^{2} \emptyset}{\partial \varphi^{2}}+k_{1}^{2} r^{2}=0 \\
\frac{r}{R}\left(\frac{\partial r}{\partial r} \cdot \frac{\partial R}{\partial r}+\frac{\partial^{2} R}{\partial r^{2}}\right)+\frac{1}{\emptyset} \cdot \frac{\partial^{2} \emptyset}{\partial \varphi^{2}}+k_{1}^{2} r^{2}=0 \\
\frac{r}{R} \cdot \frac{\partial R}{\partial r}+\frac{r^{2}}{R} \cdot \frac{\partial^{2} R}{\partial r^{2}}+\frac{1}{\emptyset} \cdot \frac{\partial^{2} \emptyset}{\partial \varphi^{2}}+k_{1}{ }^{2} r^{2}=0 \\
\quad \cdot \frac{\partial^{2} R}{\partial r^{2}}+\frac{r}{R} \cdot \frac{\partial R}{\partial r}+k_{1}{ }^{2} r^{2}=-\frac{1}{\varnothing} \cdot \frac{\partial^{2} \emptyset}{\partial \varphi^{2}}
\end{gathered}
$$

The left-hand side is a function of $r$ only, whereas the righthand side depends on $\varphi$ only. Therefore this equation can hold for all values of the variables only if both sides are equal to some constant $n^{2}$.

$$
\begin{gathered}
\frac{1}{\emptyset} \cdot \frac{\partial^{2} \emptyset}{\partial \varphi^{2}}=-n^{2} \\
\frac{\partial^{2} \emptyset}{\partial \varphi^{2}}+n^{2} \emptyset=0
\end{gathered}
$$

The solution is,

$$
\emptyset=A_{1} \cos n \varphi+A_{2} \sin n \varphi
$$

Now taking equation in r , we get,

$$
\frac{r}{R} \cdot \frac{\partial R}{\partial r}+\frac{r^{2}}{R} \cdot \frac{\partial^{2} R}{\partial r^{2}}+k_{1}^{2} r^{2}-n^{2}=0
$$

Since,

$$
\frac{1}{\emptyset} \cdot \frac{\partial^{2} \emptyset}{\partial \varphi^{2}}=-n^{2}
$$

Dividing by $r^{2}$, we get,

$$
\frac{1}{r R} \cdot \frac{\partial R}{\partial r}+\frac{1}{R} \cdot \frac{\partial^{2} R}{\partial r^{2}}+k_{1}^{2}-n^{2}=0
$$

Multiplying by R,

$$
\frac{1}{r} \cdot \frac{\partial R}{\partial r}+\frac{\partial^{2} R}{\partial r^{2}}+R\left(k_{1}^{2}-\frac{n^{2}}{r^{2}}\right)=0
$$

This is called Bessel's Equation.
Its solution is given by Bessel Function.

$$
\begin{gathered}
R=B_{1} J_{n}\left(k_{c} r\right)+B_{2} Y_{n}\left(k_{c} r\right) \\
E_{z}=R \emptyset
\end{gathered}
$$

$$
E_{z}=\left[A_{1} \cos n \varphi+A_{2} \sin n \varphi\right]\left[B_{1} J_{n}\left(k_{c} r\right)+B_{2} Y_{n}\left(k_{c} r\right)\right]
$$

Where, $J_{n}$ and $Y_{n}$ are the Bessel and Neumann function of order n , respectively. For the dominant mode of operation, the field being invariant of $\phi$ (similar to TEM mode), $\alpha$ goes to zero. The solution thus becomes

$$
E z(r)=A J_{0}\left(k_{1} r\right)+B Y_{0}\left(k_{1} r\right)
$$

With A and B as arbitrary constants.
From Maxwell's equation, we get

$$
\begin{gathered}
\nabla \times \mathrm{E}=-\mathrm{j} \boldsymbol{\omega} \mu \mathbf{H} \\
\nabla \times H=j \boldsymbol{\omega} \boldsymbol{E}
\end{gathered}
$$

So we can write

$$
\begin{gathered}
\frac{1}{r} \frac{\delta E_{z}}{\delta \phi}-\frac{\delta E_{\phi}}{\delta z}=-j \boldsymbol{\omega} \mu \boldsymbol{H}_{r} \\
\frac{\delta E_{r}}{\delta z}-\frac{\delta E_{Z}}{\delta r}=-j \boldsymbol{\omega} \mu \boldsymbol{H}_{\boldsymbol{\phi}} \\
\frac{1}{r} \frac{\delta\left(r E_{\phi}\right)}{\delta r}-\frac{1}{r} \frac{\delta E_{r}}{\delta \phi}=-j \boldsymbol{\omega} \mu \boldsymbol{H}_{z} \\
\frac{1}{r} \frac{\delta H_{z}}{\delta \phi}-\frac{\delta H_{\phi}}{\delta z}=j \boldsymbol{\omega} \boldsymbol{\epsilon} \boldsymbol{E}_{r} \\
\frac{\delta H_{r}}{\delta z}-\frac{\delta H_{Z}}{\delta r}=j \boldsymbol{\omega} \boldsymbol{\varepsilon} \boldsymbol{E}_{\boldsymbol{\phi}} \\
\frac{1}{r} \frac{\delta\left(r H_{\phi}\right)}{\delta r}-\frac{1}{r} \frac{\delta H_{r}}{\delta \phi}=j \boldsymbol{\omega} \boldsymbol{\varepsilon} \boldsymbol{E}_{z}
\end{gathered}
$$

Solving the above equations we get

$$
\begin{aligned}
E_{r} & =-\frac{j}{k_{1}^{2}}\left[\beta \frac{\delta E_{z}}{\delta r}+\frac{\omega \mu}{r} \frac{\delta H_{z}}{\delta \phi}\right] \\
E_{\phi} & =-\frac{j}{k_{1}^{2}}\left[\frac{\beta}{r} \frac{\delta E_{z}}{\delta \phi}-\omega \mu \frac{\delta H_{z}}{\delta r}\right] \\
H_{r} & =\frac{j}{k_{1}^{2}}\left[\frac{\omega \varepsilon}{r} \frac{\delta E_{z}}{\delta \phi}-\beta \frac{\delta H_{z}}{\delta r}\right] \\
H_{\phi} & =\frac{j}{k_{1}^{2}}\left[\omega \varepsilon \frac{\delta E_{z}}{\delta r}+\frac{\beta}{r} \frac{\delta H_{z}}{\delta \phi}\right]
\end{aligned}
$$

Since $\mathrm{E}_{\mathrm{Z}}$ is invariant of $\phi$ (similar to TEM mode), $\frac{\delta E_{Z}}{\delta \phi}=0$ and $\mathrm{H}_{\mathrm{z}}=0$

$$
\begin{gathered}
H_{r}=0 \\
H_{\phi}=-\frac{j}{k_{1}^{2}}\left[\omega \varepsilon \frac{\delta E_{z}}{\delta r}\right]
\end{gathered}
$$

Putting the value of $E_{z}$ and $k_{1}^{2}=k_{1}^{2}=\omega^{2} \mu \varepsilon$

$$
\begin{gathered}
H_{\phi}=-\frac{j}{\omega^{2} \mu \epsilon}\left[\omega \varepsilon \frac{\delta\left(A J_{0}\left(k_{1} r\right)+B Y_{0}\left(k_{1} r\right)\right.}{\delta r}\right] \\
H_{\phi}=-\frac{j k_{1}}{\omega \mu}\left[A J_{1}\left(k_{1} r\right)+B Y_{1}\left(k_{1} r\right)\right]
\end{gathered}
$$

## IV. Equivalent Circuit Representation

Let the modal voltage and modal current in an annular sector be defined as $\mathrm{E}_{Z}$ and $-\mathrm{rH}_{\varphi}$ respectively.[3] These definitions of the modal voltage and the modal current lead to asymmetrical Y-matrix of the structure, a necessary to be passive.
The Y-matrix is defined as

$$
\left[\begin{array}{l}
I_{1} \\
I_{2}
\end{array}\right]=\left[\begin{array}{ll}
Y_{11} & Y_{12} \\
Y_{21} & Y_{22}
\end{array}\right]\left[\begin{array}{l}
V_{1} \\
V_{2}
\end{array}\right]
$$

Where $I_{1}$ and $I_{2}$ are the line currents and $V_{1}$ and $V_{2}$ are the line voltage at ports A and B respectively.


Fig 2: An annular sector with its equivalent $\pi$ network

$$
\begin{gathered}
V_{1}=E_{z}(r), I_{1}=-r H_{\phi}(r) \\
V_{2}=E_{z}(R), I_{2}=-R H_{\phi}(R)
\end{gathered}
$$

The elements of the Y matrix are determined from the relation,

$$
Y_{11}=\frac{I_{1}}{V_{1}}\left(\text { at } V_{2}=0\right)=-\frac{r H_{\varphi}(r)}{E_{z}(r)}\left(\text { at } E_{z}(R)=0\right)
$$

Condition $E_{Z}(R)=0$, gives,

$$
\begin{gathered}
A J_{0}\left(k_{1} R\right)+B Y_{0}\left(k_{1} R\right)=0 \\
A J_{0}\left(k_{1} R\right)=-B Y_{0}\left(k_{1} R\right) \\
B / A=-\frac{J_{0}\left(k_{1} R\right)}{Y_{0}\left(k_{1} R\right)} \\
Y_{11}=-\frac{r H_{\varphi}(r)}{E_{z}(r)}=\frac{j r k_{1}\left[A J_{1}\left(k_{1} r\right)+B Y_{1}\left(k_{1} r\right)\right]}{\omega n A J_{0}\left(k_{1} r\right)+B Y_{0}\left(k_{1} r\right)}
\end{gathered}
$$

Substituting the value of $B / A$, in $Y_{11}$, we get,

$$
Y_{11}=\frac{j k_{1} r}{\omega \mu} \cdot \frac{\Delta_{1}(r, R)}{\Delta(r, R)}
$$

Where,

$$
\begin{gathered}
\Delta_{1}(r, R)=J_{1}\left(k_{1} r\right) Y_{0}\left(k_{1} R\right)-Y_{1}\left(k_{1} r\right) J_{0}\left(k_{1} R\right) \\
\Delta(r, R)=J_{0}\left(k_{1} r\right) Y_{0}\left(k_{1} R\right)-Y_{0}\left(k_{1} r\right) J_{0}\left(k_{1} R\right)
\end{gathered}
$$

Similarly $Y_{22}$ and $Y_{12}$ are obtained as

$$
\begin{aligned}
Y_{22} & =-\frac{j k_{1} R}{\omega \mu} \cdot \frac{\Delta_{1}(R, r)}{\Delta(R, r)} \\
Y_{12} & =\frac{j k_{1} R}{\omega \mu} \cdot \frac{\Delta_{1}(r, r)}{\Delta(r, R)}
\end{aligned}
$$

Now, $\Delta_{1}(r, r)$ is the Wronskian of the Bessel Equation of order zero and is equal to $2 /\left(\pi k_{1} r\right)$.Therefore, $Y_{12}$ becomes,

$$
Y_{12}=\frac{2 j}{\pi \omega \mu \Delta(r, R)}
$$

Similarly, the expression for $Y_{21}$ maybe obtained. This shows that $Y_{21}=Y_{12}$. Thus reciprocity holds, justifying the definitions of the modal voltage and the modal current used. The Y matrix of the other section can be obtained from the Y matrix of the earlier one and its element values are given


Fig 3: Equivalent network when two annular sectors are cascaded

$$
\begin{gathered}
Y_{11}=\frac{I_{1}}{V_{1}}\left(\text { at } V_{2}=0\right) \\
Y_{11}{ }^{\prime}=-\frac{I_{2}}{V_{2}}\left(\text { at } V_{1}=0\right)=-Y_{22} \\
Y_{22}=\frac{I_{2}}{V_{2}}\left(\text { at } V_{1}=0\right) \\
Y_{22}{ }^{\prime}=-\frac{I_{1}}{V_{1}}\left(\text { at } V_{2}=0\right)=-Y_{11} \\
Y_{12}=\frac{I_{1}}{V_{2}}\left(\text { at } V_{1}=0\right) \\
Y_{12}^{\prime}=-\frac{I_{2}}{V_{1}}\left(\text { at } V_{2}=0\right)=-Y_{21}=-Y_{12} \\
Y_{21}=\frac{I_{2}}{V_{1}}\left(\text { at } V_{2}=0\right) \\
Y_{21}{ }^{\prime}=-\frac{I_{1}}{V_{2}}\left(\text { at } V_{1}=0\right)=-Y_{12}
\end{gathered}
$$

Using the Y matrices of the two sections, the ABCD parameters of the unit cell are obtained.
For section (1),

$$
\left[\begin{array}{ll}
A & B \\
C & D
\end{array}\right]=\left[\begin{array}{cc}
\frac{-Y_{22}}{Y_{21}} & \frac{1}{Y_{21}} \\
\frac{-\left(Y_{11} \cdot Y_{22}-Y_{12} \cdot Y_{21}\right)}{Y_{21}} & \frac{Y_{11}}{Y_{21}}
\end{array}\right]
$$

For section (2),

$$
\left[\begin{array}{ll}
A^{\prime} & B^{\prime} \\
C^{\prime} & D^{\prime}
\end{array}\right]=\left[\begin{array}{cc}
\frac{-Y_{11}}{Y_{12}} & -\frac{1}{Y_{12}} \\
\frac{\left(Y_{11} \cdot Y_{22}-Y_{12} \cdot Y_{12}\right)}{Y_{12}} & \frac{Y_{22}}{Y_{12}}
\end{array}\right]
$$

The overall ABCD matrix is obtained by cascading two sections,

$$
\left[\begin{array}{ll}
A & B \\
C & D
\end{array}\right]=\left[\begin{array}{cc}
2 \frac{Y_{22} Y_{11}}{Y_{12}^{2}}-1 & 2 \frac{Y_{22}}{Y_{12}^{2}} \\
2 \frac{Y_{11}}{Y_{12}}\left(\frac{Y_{11} Y_{22}}{Y_{12}}-Y_{12}\right) & 2 \frac{Y_{22} Y_{11}}{Y_{12}^{2}}-1
\end{array}\right]
$$

The S-matrix is obtained from the ABCD matrix.

$$
\begin{aligned}
S_{11} & =\frac{Z_{0} A+B-Z_{0}^{2} C-Z_{0} D}{Z_{0} A+B+Z_{0}^{2} C+Z_{0} D} \\
S_{12} & =\frac{2 Z_{0}(A D-B C)}{Z_{0} A+B+Z_{0}^{2} C+Z_{0} D} \\
S_{21} & =\frac{2 Z_{0}}{Z_{0} A+B+Z_{0}^{2} C+Z_{0} D} \\
S_{22} & =\frac{-Z_{0} A+B-Z_{0}^{2} C+Z_{0} D}{Z_{0} A+B+Z_{0}^{2} C+Z_{0} D}
\end{aligned}
$$

From the S -matrix the elements of the WAT matrix are obtained as,

$$
\begin{gathered}
A_{11}=\frac{1}{S_{12}}=\frac{1}{2}\left[A+\frac{B}{Z_{0}}+Z_{0} C+D\right] \\
A_{12}=\frac{-S_{22}}{S_{12}}=\frac{1}{2}\left[A-\frac{B}{Z_{0}}+Z_{0} C-D\right] \\
A_{21}=\frac{S_{11}}{S_{12}}=\frac{1}{2}\left[A+\frac{B}{Z_{0}}-Z_{0} C-D\right] \\
A_{22}=\frac{S_{12} \cdot S_{21}-S_{11} S_{22}}{S_{12}}=\frac{1}{2}\left[A-\frac{B}{Z_{0}}-Z_{0} C+D\right]
\end{gathered}
$$

Where in the above equation $Z_{0}$ is the characteristic impedance of the terminating line and is given by $Z_{0}=\frac{120 \pi}{r \sqrt{\epsilon_{r}}}$ . It may be noted that the terminating line is assumed to have the width a.

## A) Propagation Constant

In a periodically loaded structure, an infinite number of special harmonics is inevitably and simultaneously present with varying propagation constants. The propagation constant [2] of the space harmonic $\beta_{0}$ is obtained as

$$
\beta_{0}=\frac{1}{p} \arccos A
$$

Since A is always real and therefore $\beta_{0}$ is also really only when $|A|<1$. substituting the expression for A , the condition for propagation becomes as given below

$$
0<\frac{Y_{11} Y_{22}}{Y_{12}^{2}}<1
$$

Or

$$
0<\left[\frac{\pi^{2} k_{1}^{2} R r}{4} \Delta_{1}(R, r) \Delta_{1}(r, R)\right]<1
$$

The term inside the bracket in the previous equation is dependent on frequency. Thus, frequencies for which the previous equation is not satisfied lie in the stop-band. Any imaginary value for $\beta_{0}$ can be considered as stop-band.

## B) Characteristic Impedance

The characteristic impedance is another important parameter to be studied. The normalized characteristic impedance $Z_{0}{ }^{\prime}$ [2] is obtained as

$$
Z_{0}^{\prime}=\frac{2 A_{12}-A_{11}+A_{22} \pm\left[\left(A_{11}+A_{22}\right)^{2}-4\right]^{\frac{1}{2}}}{2 A_{12}+A_{11}-A_{22} \mp\left[\left(A_{11}+A_{22}\right)^{2}-4\right]^{\frac{1}{2}}}
$$

Substituting the values of WAT matrix parameter in terms of ABCD parameters

$$
Z_{0}^{\prime}=\frac{\sqrt{\left(A^{2}-1\right)}-B / Z_{0}}{C Z_{0}-\sqrt{\left(A^{2}-1\right)}}
$$

Where in the above equation $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ are calculated before and $z_{0}=\frac{120 \pi}{r \sqrt{\epsilon_{r}}}$

## V. Design of Triangular Periodic Microstrip Transmission Line

The equations obtained in the field analysis are simulated using MATLAB software for further study. This is done for the easier calculation of design parameters and reduces the computation time reasonably.


Fig 4: Propagation constant $\beta_{0}$ vs Frequency plot
In the frequency range from 8 GHz to 12 GHz , probable stopbands are obtained where the value of $\beta_{0}$ p is nearly equal to zero.


Fig 5: Fabricated Microstrip Line

$\mathrm{a}=0.22 \mathrm{~cm}, \mathrm{~b}=0.48 \mathrm{~cm}, \mathrm{p}=1 \mathrm{~cm}$ and $\mathrm{h}=0.08 \mathrm{~cm}$


Fig 6: Experimental setup Fabricated Microstrip Line

## VI. Result Analisis

To verify the range of frequency mapped on the time base of the CRO, from the klystron power supply, another experimental set-up is used where the microstrip transmission line has not been connected. The set-up is shown above and below. From this experiment, it is measured that the waveform range of frequency mapped can be measured as 9.55 GHz to 9 . 601 GHz .The frequency is measured with the help of a frequency meter. So, with this set-up, after connecting the microstrip line we can study the passband and stopband only in the range.


Fig 7: Block diagram representation of experimental
We also study the nature of wave propagation through the microstrip line we have used the experimental set-up shown in Fig: 7. As there is a drop in the signal near the frequency 9.57 GHz , there is a stop-band at frequency 9.57 GHz .

## VII. Conclusion

The intrinsic properties of the triangularly width modulated periodic structure can be used in the design and the construction of some useful microwave components such as filters. Periodically loaded line exhibits alternating bands of propagation and attenuation, making it possible to realize a bandpass filter or a band-stop filter with an infinite number of passbands. The existence of the passband and the stopband depends on the value of the propagation constant. If for a certain band of frequencies, the value of propagation constant becomes imaginary, then that band of frequencies can be indicated as stopband. The position and width of a passband or a stopband can be controlled by adjusting the design parameters of the periodic structure such as $b / a$ ratio and periodicity. This type of microstrip line is used extensively because they provide a free and accessible surface on which solid-state devices can be placed. The open strip provides better interconnection features. They are used extensively to interconnect high-speed logic circuits in digital computers because they can provide the required uniform signal paths.

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