# Analysis of the Adequacy of L. Euler'S Critical Parameters to the Results of Calculations of Systems for Loss of Stability of the Form

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# Analysis Of The Adequacy Of L. Euler's Critical Parameters To The Results Of Calculations Of Systems For Loss Of Stability Of The Form

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**Abstract.** It is obvious that in modern conditions, information technologies are being intensively introduced in all spheres of human activity, allowing for a comprehensive systematic approach to all aspects of modern human life. Of course, the technologies being implemented before their practical use are first conceived, developed and formalized by their algorithmization. The paper is devoted to the formalized algorithmization of structural analysis processes to reliability requirements in terms of their resistance to formally possible manifestations of critical impacts during the operation of buildings and structures. The algorithm of structural stability analysis considered in this paper consists of many separate different steps, and this set is finite. The principle of discreteness is observed in the analysis algorithm. A descriptive algorithm compiled in natural and mathematical languages is proposed.

## INTRODUCTION

The phenomenon of loss of stability can occur in many cases and in any structures. And in plates, and in shells, and in rod systems. Thus, it is obvious that stability as a phenomenon for both simple systems and complex structures consisting of many elements depends on the stability of individual elements and on their joint work in structures. The loss of stability in many cases can be characterized as a bifurcation[1] of the equilibrium form. For example, a rectilinear rod, when the longitudinal force reaches a critical value, can also take a curved shape, i.e. lose stability.

Two approaches are usually used to establish the equilibrium of the system: 1) the principle of possible displacements, in which: if the system is in equilibrium, the sum of all external and internal forces in any infinitely small displacements equal to zero. 2) the property of the potential energy of the system, which: if the system is in equilibrium, its potential energy (i.e. the energy external and internal forces) has an extreme value.

However, these methods, as signs, do not answer the question whether the equilibrium is stable or unstable. It is known that the Lagrange-Dirichlet principle can answer this question: the equilibrium of a system is stable if its total potential energy is minimal compared to all sufficiently close positions of the system.

In the future, approach №1 will be used in the work to eliminate the problems that have arisen.

## MATERIALS AND METHODS

### About definitions.

Let's define the concept of "stability" of buildings and structures by the ability of their elements to maintain their original geometric dimensions at a constant level throughout the entire service life, despite aggressive environmental influences

At the same time, we will assume that these positions or forms of equilibrium in working condition during operation throughout the entire life cycle will be considered stable if, under any influences, structural elements deforming from these influences, nevertheless retain properties to restore their geometric dimensions and operational properties.

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1 Bifurcation – splitting, doubling.

Buildings and structures, their elements, as systems that do not possess these properties, are subject to destructive states with a high degree of reliability of a priori statistics, and natural destruction can occur with them throughout the life cycle, both of individual elements and structures as a whole.

Thus, the constructions considered by us, as systems, are equally likely to be in different states from the point of view of estimating geometric parameters and properties in the dynamics of their states. But in time, they can alternate between both equilibrium and non-equilibrium states. The period of time and the state of the properties of systems at such transitional moments of time will be called "loss of stability", and this moment in time itself will be called the "critical state" of the object.

At the same time, the modules of the magnitudes of the applied external loads causing such states are called critical forces.

On the calculation of the stability of frame structures.

To calculate stability, the so-called critical loads "RCR" are found using L. Euler's multifunctional critical parameter "v". The values of this parameter depend on the adopted main system, the type and form of the applied forces, on the types of connections of the structural elements with each other. In addition, to determine the "RCR" at the initial stage of solving the problem, the so-called coefficient of the calculated length "µ" is also found.

Thus, the calculation for the stability of systems consists in determining the critical parameter "v", according to which the values of the critical loads "RCR" are further determined based on the fact that:

$$P_{\text{кр}} = \frac{v^2 \cdot E \cdot I}{l^2} \tag{1}$$

The algorithm for calculating frame structures for stability.

Consider the method of calculating the above elements and systems for stability. At the same time, we will use both calculated and tabular and hardware forms and values of the reactive behavior of structures, both for real and virtual external influences, while using appropriate normative literary applications.

## Preparation of the calculation scheme

Let's formulate the algorithm of the calculation method:

1. First, it is necessary to conduct a kinematic analysis of the design scheme according to the formula:

$$n = n_a + n_L = n_a + (2 \cdot N - C - C_0) \tag{2}$$

where $n_a$– the number of rigid nodes in the structure: $n_L$- the number of possible linear deformations of the nodes of the structure; N – the number of nodes of the structure; C- number of rods in the structure; $C_0$- the number of hinge supports.

1.1 At the next stage, based on the calculation scheme, we will develop the so-called "main system" of the system under consideration, thereby constructing and designing this structure as a geometrically immutable system.

2. We will determine the parameters of the system that potentially have the ability to bring it to a previously defined state of "loss of stability" using the following ratio:

$$v_i = k \cdot l_i = \sqrt{\frac{P_i}{E \cdot I_i}} \tag{3}$$

where: $v_i$- «critical parameter»; k- proportionality coefficient; $l_i$- length of the structure; $P_i$- load; $E \cdot I_i$- stiffness of the section.

Based on the results obtained, we will choose such an element of the design under consideration, in which the parameter "v" will receive the maximum value from the considered values.

After finding the specified parameter, the "critical" parameters of other elements will be correlated to the maximum value of the found parameter.

3. In an analytical form, the equilibrium state of the considered construct can be represented by a system of canonical equations without free terms. Why? Because the forces acting in the direction coinciding with the geometric axis of the elements do not create bending moments for these elements. Thus:

$$\begin{cases} r_{11} \cdot Z_1 + r_{12} \cdot Z_2 + \cdots + r_{1n} \cdot Z_n = 0 \\ r_{21} \cdot Z_1 + r_{22} \cdot Z_2 + \cdots + r_{2n} \cdot Z_n = 0 \\ \qquad\qquad \ldots \\ r_{n1} \cdot Z_1 + r_{n2} \cdot Z_2 + \cdots + r_{nn} \cdot Z_n = 0 \end{cases} \tag{4}$$

At the next stage, we will construct "single plots" of bending moments Mi in the accepted basic system of the design scheme from the action of single torsional and linear virtual loads. At the same time, we use both literary and empirical forms of their representation

5. After that, we determine the values of the coefficients for the unknown Zi using the above system of canonical equations. Using the values of $r_{ij}$ found from the above system, we then compose the so-called matrix equation of the stability of the system in the following form:

$$M = \begin{vmatrix} r_{11}r_{12} \, ... \, r_{1n} \\ r_{21}r_{22} \, ... \, r_{2n} \\ ... \\ r_{n1}r_{n2} \, ... \, r_{nn} \end{vmatrix} = 0 \tag{5}$$

Let's pay attention to the fact that in the problems with strength determinant of the matrix M often $> 0$, and in stability problems, determinant of the matrix M usually $= 0$.

6. To solve the specified matrix equation with respect to the parameter "v", as a rule, iterative solution methods are used. In this case, the initial values of the parameter "v" will be set based on the conditions of interaction of the structural elements with each other.

7. To determine the "critical forces" we will use the following relation:

$$P_{\text{кр}} = \frac{v^{2} \cdot EI}{l^{2}} \tag{6}$$

8. The values of the reduced lengths of the elements will be further calculated using the following formula:

$$l_0 = \mu \cdot l = \frac{\pi}{v} \cdot l \tag{7}$$

The values of $\mu$ and v will be taken, taking into account calculations, and l - based on the given geometry of the structure.
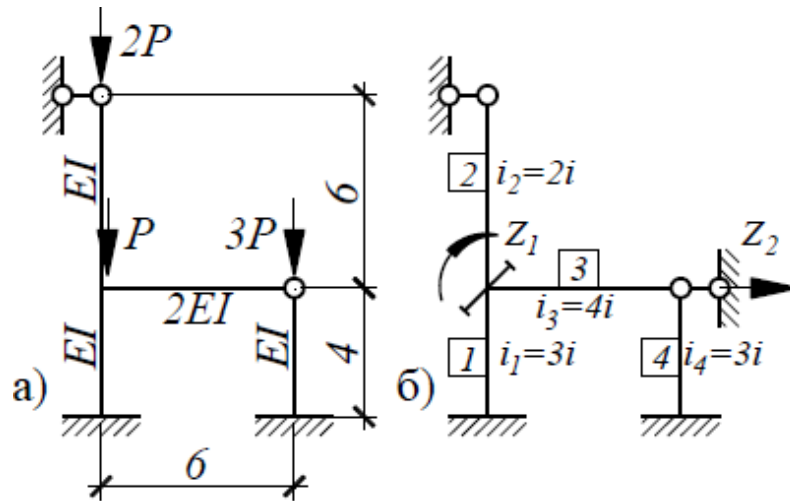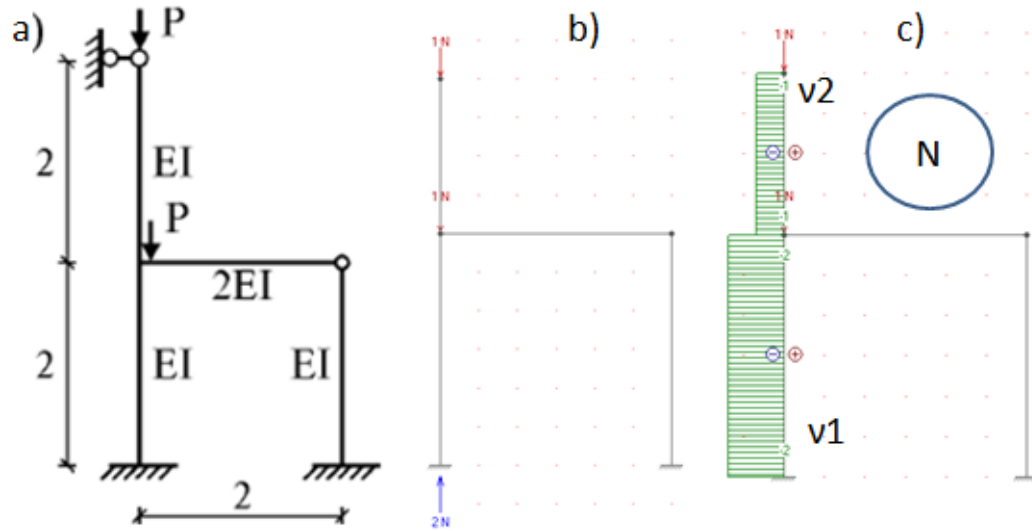
## DISCUSSION AND RESULTS



**FIGURE 1.** Design scheme of the analyzed structure

Let's illustrate the above with the following actions. Suppose, for the calculation scheme given in Fig. 1, it is necessary to determine the critical forces of the **Pcr**.
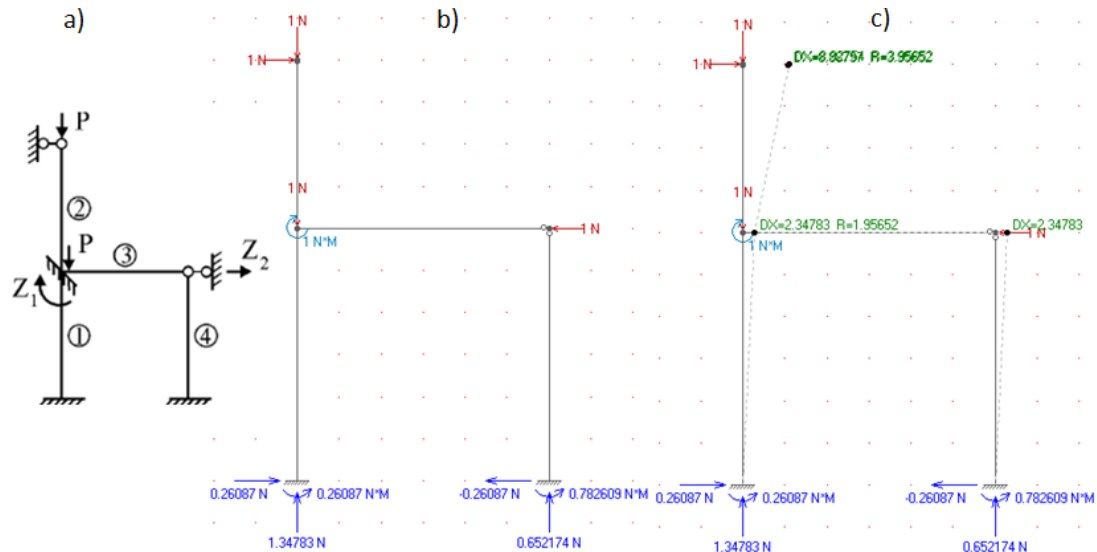
**FIGURE 2.**
a-design scheme; b-main system; c-longitudinal forces plot; "v1.2"-critical Euler parameters

For the calculation scheme shown in Fig.2a, we propose the basic system given in the same place, scheme (b). For it, we determined the internal forces M, Q and N, having previously determined its reference reactions. Fig. 2 "c" shows that in this case M and Q are zero.. Then it is obvious that only longitudinal forces N are present in the elements of the analyzed structure from the applied vertical loads. Then here you need to ask the question: -"How adequately does the specified system comply with the provisions on immutable systems?" Then we will further evaluate this system by analyzing its kinematic stability. Let's determine the number of degrees of freedom of this system: $n = n_a + n_L = 1 + 1 = 2$

It turns out that the analyzed construction is twice statically indeterminate. Optimization of this system consists in finding the so-called "superfluous connections", as well as in finding and minimizing the values of its values M, Q and N. The main system and the "balancing" of the structure by angular and linear reactions of the bonds $Z_1$ and $Z_2$, respectively, are shown in Fig.3 (a, b).



**FIGURE 3.** Deformation scheme of the main system of the analyzed structure. a-the main system; b-the virtual scheme of the application of "balancing" forces compensating for possible displacements; c- deformation scheme of possible deformations;

Using the "Lagrange principle" of possible displacements, the values of possible displacements are calculated and presented in Fig.3. Along with this, we have determined the distributions of internal forces M,Q and N, from possible deformations in the form of corresponding plots, Fig.4 (a,b,c).
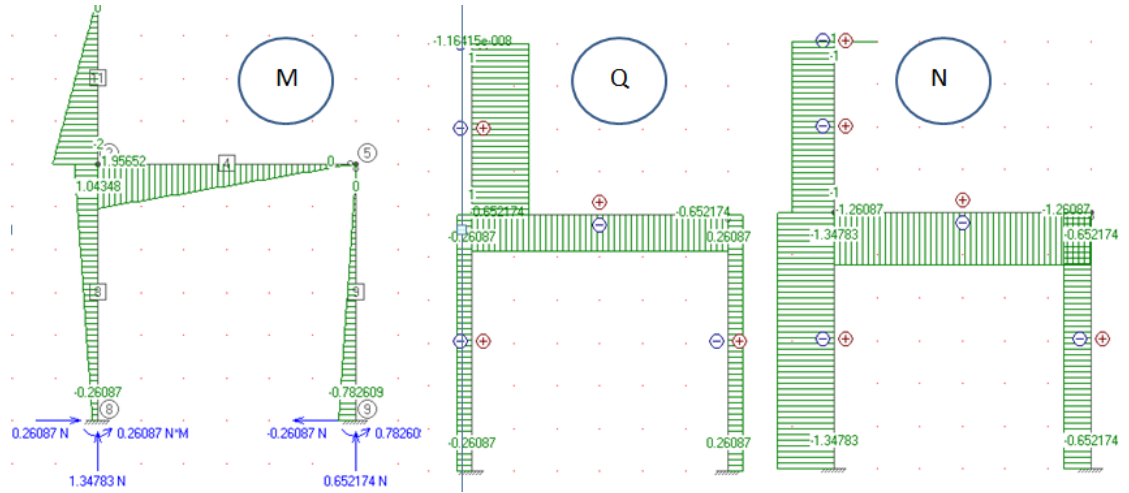
**FIGURE 4.** Distributions of internal forces M, Q, and N

Comparison of Fig.2 and Fig.3 clearly indicates significant changes in the distribution of internal efforts in the first and second cases. Let's pay attention to the fact that in the construction, at the same time, internal stresses (M, Q and N) appeared,. This fact indicates that a stress-strain state has arisen in the structural elements, initiated by virtual deformations.

Let us consider in Fig.4 two states of the main system obtained as a result of the use of virtual single reactive bonds $Z_1$ and $Z_2$, These bonds virtually compensate for the two degrees of freedom that we found earlier, one angular degree of freedom and one linear degree of freedom
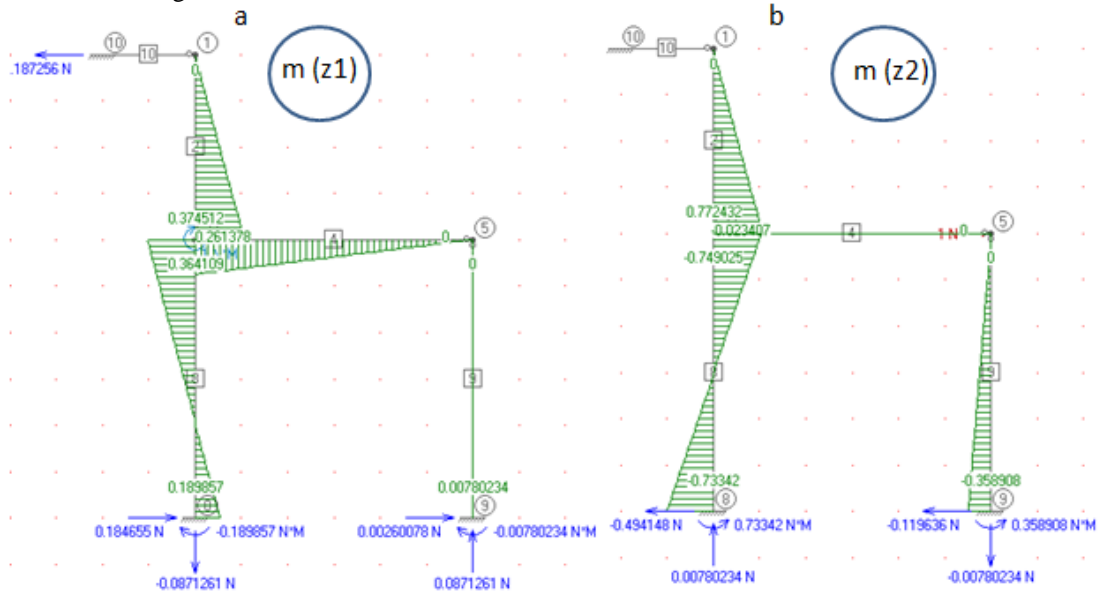


**FIGURE 5.** Single plots of moments from reactive bonds $Z_1$ and $Z_2$.
a-from $Z_1$; b- from $Z_2$

Considering the analyzed structure from the point of view of analyzing the possible loss of stability and noting the nonlinearity of the values of factors that can potentially lead the system to such a result, we point out that in this case, the graphs of moments in Fig.5 - must be non-linear.

We will calculate the critical stability parameters of L. Euler for the elements of the analyzed structure using the following relations (8):

$$v_1 = 2 \cdot \sqrt{\frac{2 \cdot P}{E \cdot I}}, \quad v_2 = 2 \cdot \sqrt{\frac{P}{E \cdot I}}, \quad v_3 = v_4 = 0 \tag{8}$$

Taking into account the calculation below, the largest value of the stability parameter of the considered elements belongs to the first core of the main system. Based on this, for further calculations we will select the parameter with the highest value as the main critical parameters and, - denote it as $v_1$.

$$If \ v_1 = 2 \cdot \sqrt{\frac{2 \cdot P}{E \cdot I}} = v \ , \ then \ \frac{v_2}{v} = \frac{2 \cdot \sqrt{\frac{\cdot P}{E \cdot I}}}{2 \cdot \sqrt{\frac{2 \cdot P}{E \cdot I}}} = \frac{1}{\sqrt{2}} \approx 0,7, \ - \ hence \ v_2 = 0,7v$$

At the next stage of the algorithm implementation , we will compile a system of canonical equations for the considered case of loading of the main system and the reaction of the system to this loading , in which $R_{1P} = R_{2P} = 0$:

$$\begin{cases} r_{11} \cdot Z_1 + r_{12} \cdot Z_2 = 0 \\ r_{21} \cdot Z_1 + r_{22} \cdot Z_2 = 0 \end{cases} \tag{9}$$

The reduced homogeneous system of equations can have many solutions, and in particular we show some of them:

$$If \ D = det \begin{vmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{vmatrix} = 0 \ provided \ that \ r_{12} = r_{21} \ then$$

$$D(v) = r_{11} \cdot r_{22} - r_{12}^2 \tag{10}$$

The equation (10) shown by us can be interpreted as a formalization of the equilibrium condition of the analyzed structure from the impact of critical loads, expressed in mathematical language (the stability equation).

Let's continue the search for unknown coefficients of the system of canonical equations (9) for the subsequent determination of the values of the quantities $Z_1$ and $Z_2$, To determine the coefficients of the system, we will use, among other things, the data shown in Fig.4a in analytical form:

$$r_{11} = 4 \cdot \frac{E \cdot I}{2} \cdot \varphi_1(v_1) + 3 \cdot \frac{E \cdot I}{2} \cdot \varphi_5(v_2) = E \cdot I[2 \cdot \varphi_1(v_1) + 1,5 \cdot \varphi_5(v_2) + 3]$$

$$r_{12} = -6 \cdot \frac{E \cdot I}{2^2} \cdot \varphi_3(v_1) + 3 \cdot \frac{E \cdot I}{2^2} \cdot \varphi_5(v_2) = E \cdot I[-1,5 \cdot \varphi_3(v_1) + 0,75 \cdot \varphi_5(v_2)]$$

$$r_{22} = 12 \cdot \frac{E \cdot I}{2^3} \cdot \varphi_4(v_1) + 3 \cdot \frac{E \cdot I}{2^3} \cdot \varphi_6(v_2) + 3 \cdot \frac{E \cdot I}{2^3} = E \cdot I[1.5 \cdot \varphi_4(v_1) + 0,375 \cdot \varphi_6(v_2) + 375]$$

How to find the roots of the stability equation (10). To do this, we will substitute the previously found coefficients of the system into the stability equation. Next, we choose from all the roots we found, the root value, which will be the smallest value of the root of the stability equation. To determine the smallest positive root of this equation, we first look for the range of possible values in which this root of the equation can be located.

Let us take into account all the previously given arguments, as well as all the previously given analytical dependencies, then on their basis we can draw the following conclusion. The stability of the analyzed structure directly depends on the operation of element No. 1, taking into account the value of the non-linear and multi-parametric critical parameter of L. Euler. Then it is obvious that : $v_1 = v$,

Therefore, in the future, based on the analysis of the work of element No. 1, we will be able to judge the work of the entire structure as a whole.

Let's take a closer look at the work of element No. 1 separately from the work of other structural elements by applying the "superposition principle" in the analysis of the work of this structure (Fig. 5 a, b).

Then, in the case of the operation of element No. 1 in accordance with option (a), as shown in Fig. 5: $\mu = 2$ $v = \frac{\pi}{\mu} = \frac{3,14}{2} = 1,57$.

Then, in the case of the operation of element No. 1 in accordance with option (b), as shown in Fig. 5: $\mu = 0,5$ $v = \frac{\pi}{\mu} = \frac{3,14}{0,5} = 6,28$
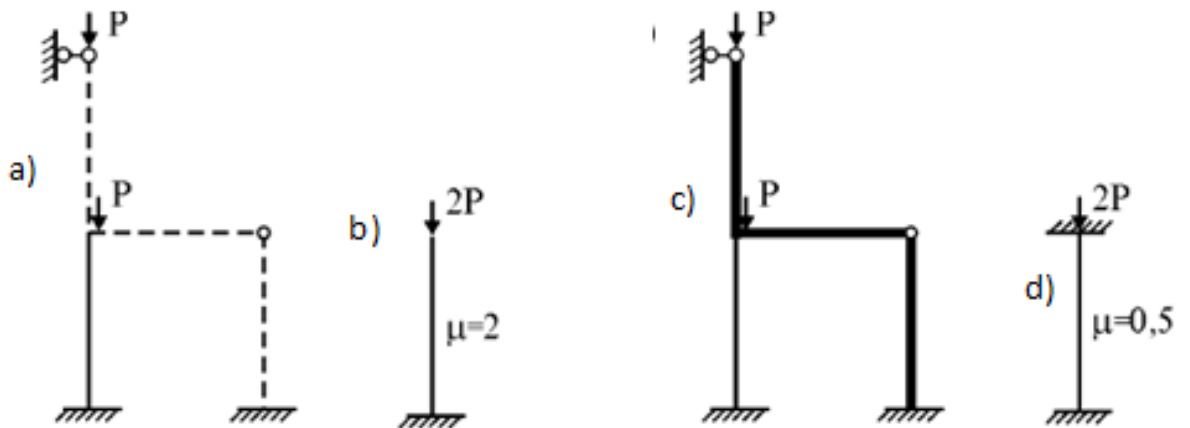


**FIGURE 6.** Justification of the values of $\mu_i$ for finding the interval of values of the critical parameter $v_j$

If the critical parameter for element No. 1 with the same probability for all other elements of the scheme can take values in the range $from$ $0$ $to$ $\infty$ then the root of the stability equation must be in the interval:

$$1{,}57 \leq v_j \leq 6{,}28$$

Solution of the stability equation by the method of successive approximations for the tabulated values of the parameter $\varphi_i(v_j)$ is summarized in table 1:

**TABLE 1.** Relations between the critical parameters of L. Euler and the parameter D of the stability equation

| v=v1 | v2 | φ1(v1) | φ3(v1) | φ4(v1) | φ5(v2) | φ6(v2) | r11 | r12=r21 | r22 | D |
|------|------|--------|--------|--------|--------|--------|-------|---------|-------|--------|
| 2,8 | 2,0 | 0,706 | 0,861 | 0,208 | 0,294 | 2,317 | 1,154 | 0,338 | 1,38 | -1,706 |
| 4,0 | 2,8 | 0,293 | 0,696 | 0,637 | 2,173 | 7,506 | 9,806 | 4,154 | 0,631 | 23,446 |
| 4,9 | 3,4 | 0,361 | 0,505 | 1,495 | 4,146 | 3,857 | 3,964 | 1,601 | 3,806 | 17,649 |
| 5,7 | 4,0 | -2,18 | 0,258 | -2,45 | 1,124 | 9,707 | 1,361 | 0,387 | 2,895 | 3,79 |

When solving the stability equation, including using Table 1, in the found interval $1{,}57 \leq v_j \leq 6{,}28$, we determine the values of $v_j$, between which the function D changes its sign. So, for $v_1 = 5{,}9$ it turns out that *D=3,790 > 0, and for* $v_1 = 4{,}9$ *D=-17,649 < 0.* This means that the critical parameter, as the root of the stability equation, is between the values of 4.9 and 5.9.

Sequentially reducing the interval between the values of $v_j$ with different signs, as a result we get the table.2:

**TABLE 2.** Relations between the critical parameters of L. Euler and the parameter D of the stability equation

| v=v1 | v2 | φ1(v1) | φ3(v1) | φ4(v1) | φ5(v2) | φ6(v2) | r11 | r12=r21 | r22 | D |
|------|------|--------|--------|--------|--------|--------|-------|---------|-------|--------|
| 2,8 | 2,0 | 0,706 | 0,861 | 0,208 | 0,294 | 2,317 | 1,154 | -0,338 | 1,380 | -1,706 |
| 4,0 | 2,8 | 0,293 | 0,696 | 0,637 | 2,173 | 7,506 | 9,806 | 4,154 | 0,631 | 23,446 |
| 4,9 | 3,4 | 0,361 | 0,505 | 1,495 | 4,146 | 3,857 | 3,964 | 1,601 | 3,806 | 17,649 |
| 5,7 | 4,0 | 2,180 | 0,258 | 2,450 | 1,124 | 9,707 | 1,361 | 0,387 | 2,895 | 3,790 |
| 5,3 | 3,7 | 0,942 | 0,394 | 1,947 | 2,067 | 7,297 | 1,116 | 0,590 | 1,038 | -1,507 |
| 5,5 | 3,9 | 1,418 | 0,329 | 2,192 | 1,546 | 8,538 | 0,164 | 0,494 | 1,752 | -0,530 |
| 5,6 | 3,9 | 1,748 | 0,294 | 2,319 | 1,327 | 9,127 | 0,496 | 0,442 | 2,247 | 0,920 |

From the table.2 we find the maximally reduced interval for the function D, which in our case is: $-0{,}530 \leq D_i \leq 0{,}920$. These values of the function D correspond to the minimum range of values of the critical parameter of L. Euler: $5{,}5 \leq v_1 \leq 5{,}6$.

We interpolate the specified interval using a linear proportion:

$$\frac{0{,}920-(-0{,}530)}{5{,}6-5{,}5} = \frac{100}{X} \qquad where \ from \qquad X = 25{,}663.$$

Since the value of the found number is specified after the second decimal place, let's take the value of the specified number equal to $v_1 = 5{,}574$. Then $v_2 = 0{,}7 \cdot v_1 = 3{,}902$.

Further, it is for these values of the critical parameter found by us that we calculate the so-called "transformed" length $l_0$ of the elements of the structure, as well as the critical force, using the following relations:

$$\mu_k = \frac{\pi}{v_j} \qquad и \qquad P_{m \ cr} = \frac{v_{j \ cr}^2}{l_m^2} \cdot E \cdot I \tag{11}$$

The results of the calculations are presented as follows:

Thus, the "converted" length is $l_{01}$: $\mu_1 = \frac{\pi}{v_1} = \frac{3{,}14}{5{,}574} = 0{,}563$; $l_{01} = \mu \cdot l = \frac{\pi}{v} \cdot l = 0{,}563 \cdot 2 = 1{,}126$;

Thus, the "converted" length is $l_{02}$: $\mu_2 = \frac{\pi}{v_2} = \frac{3{,}14}{3{,}902} = 0{,}805$; $l_{02} = \mu \cdot l = \frac{\pi}{v} \cdot l = 0{,}805 \cdot 2 = 1{,}610$;

Critical power: $P_{1\ cr} = \frac{v_1^2}{l_1^2} \cdot E \cdot I = \frac{31,073}{4} \cdot E \cdot I = 7,768 \cdot E \cdot I;$

Critical power: $P_{2\ cr} = \frac{v_2^2}{l_2^2} \cdot E \cdot I = \frac{15,226}{4} \cdot E \cdot I = 3,806 \cdot E \cdot I$

The influence of the argument $v_1$ on the function $D(v_1)$ in the work is estimated by a polynomial of the sixth degree with a degree of adequacy $R^2=1$.

$$D(v1) = -0,3057(v1)^6 + 7,301(v1)^5 - 67,993(v1)^4 + 310,37(v1)^3 - 713,68(v1)^2 + 759,56(v1)$$
$$2,8 < (v1) < 6,5 \tag{12}$$

The influence of the argument $v_2$ on the function $D(v_2)$ in the work is also estimated by a polynomial of the sixth degree with a degree of adequacy $R^2=0,9606$.

$$D(v2) = -0,0994(v2)^6 + 2,302(v2)^5 - 20,23(v2)^4 + 82,411(v2)^3 - 148,6(v2)^2 + 83,121(v2)$$
$$2,0 < (v1) < 4,6 \tag{13}$$

The relationship between the calculated arguments ($v_1$, $v_2$) and the function $D(v_1, v_2)$, as response functions are shown in figure 6 as a response surface from the specified arguments.
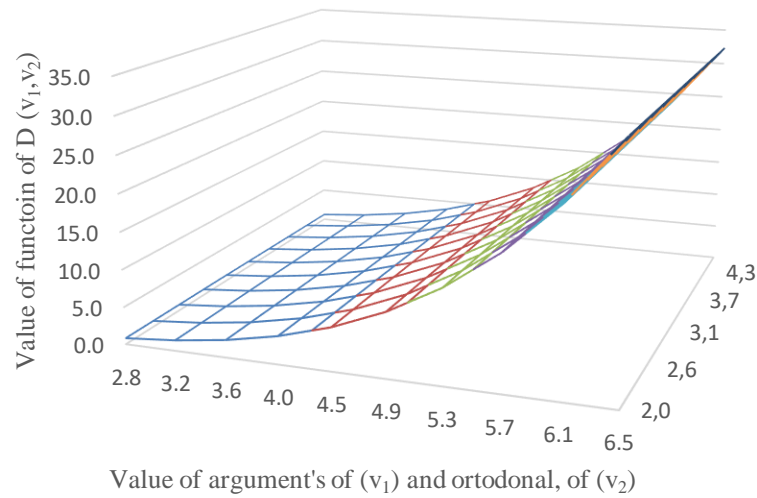


**FIGURE 7** Relations between the critical parameters of L. Euler $v_1$,$v_2$ and the parameter of $D(v_1,v_2)$ of the stability equation (10).

The behavior of the function $D(v_1, v_2)$, which is a condition for the stability of the analyzed system, as follows from figure 6, is non-linearly influenced with varying intensity by both arguments $v_1$ and $v_2$. Therefore, in further calculations for the stability of the analyzed system, both of these impacts are taken into account. In particular, the roots of the above polynomials are determined, i.e., the zeros of the function $D(v_1, v_2)$. So for the polynomial (12) there are six roots, four real and two complex conjugate:$v1(1 \div 6) \simeq 4,88; 3,71; 6,97; 6,06; 1,12 + 0,044i; 1,12 - 0,044i.$.

To check the correctness of the choice of confidence intervals $2,8 < (v1) < 6,5$ и$2,0 < (v1) < 4,6$ local extremes of the function are determined $D(v_1, v_2)$. Algorithmic process of finding extreme values $D(v_1)$ in the interval $2,8 < (v1) < 6,5$ shown on figure 7:
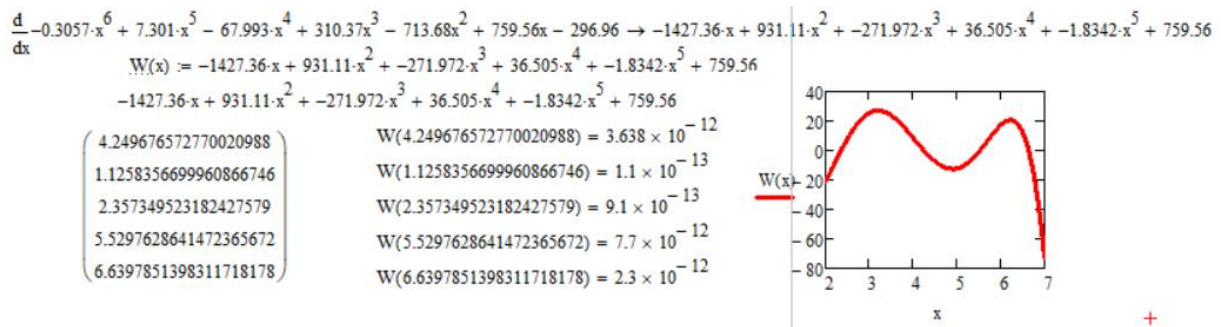


**FIGURE 8.** Algorithmic process of finding extreme values of $D(v_1)$

The calculated extreme values were recoded. The value of the value $D(v_1)$ corresponds to the value W(x), and the value $v_1$ corresponds to the value X. Figure 7 shows the local extreme values of the polynomial dependence of the function D on the argument $v_1$ in recoded form. The dynamics of the nonlinear influence of parameter v1 on local extreme values is shown on the corresponding graph in the same place.

Algorithmic process of finding extreme values of $D(v_2)$ in the interval $2,0 < (v1) < 4,6$ shown on figure 8. The same as for the function $D(v_1)$, for the function $D(v_2)$ transcoding has been performed. So, the value of R(x) corresponds to the value $D(v_2)$, and the size of $v_2$ – value X. The dynamics of the nonlinear effect of parameter v2 on local extreme values is shown on the corresponding graph in the same place.

$$\frac{d}{dx}\left(-0.0994 \cdot x^6 + 2.3026 \cdot x^5 - 20.231 \cdot x^4 + 52.411x^3 - 148.6x^2 + 83.121x\right) \rightarrow -297.2 \cdot x + 157.233 \cdot x^2 + -80.924 \cdot x^3 + 11.513 \cdot x^4 + -0.5964 \cdot x^5 + 83.121$$

$$R(x) := -297.2 \cdot x + 157.233 \cdot x^2 + -80.924 \cdot x^3 + 11.513 \cdot x^4 + -0.5964 \cdot x^5 + 83.121$$

$$-297.2 \cdot x + 157.233 \cdot x^2 + -80.924 \cdot x^3 + 11.513 \cdot x^4 + -0.5964 \cdot x^5 + 83.121$$

$$\begin{pmatrix} 0.32722494953037162102 \\ 0.73555293500512701992 - 1.9344407937085831348i \\ 0.73555293500512701992 + 1.9344407937085831348i \\ 8.7529137317454484036 - 4.7778874399323580544i \\ 8.7529137317454484036 + 4.7778874399323580544i \end{pmatrix}$$

$R(0.32722494953037162102) = -1.4 \times 10^{-14}$

$R(0.73555293500512701992 - 1.9344407937085831348i) = 4.3 \times 10^{-14} - 8.9i \times 10^{-14}$

$R(0.73555293500512701992 + 1.9344407937085831348i) = 4.3 \times 10^{-14} + 8.9i \times 10^{-14}$

$R(8.7529137317454484036 - 4.7778874399323580544i) = -1.4 \times 10^{-11} - 7.3i \times 10^{-12}$

$R(8.7529137317454484036 + 4.7778874399323580544i) = -1.4 \times 10^{-11} + 7.3i \times 10^{-12}$
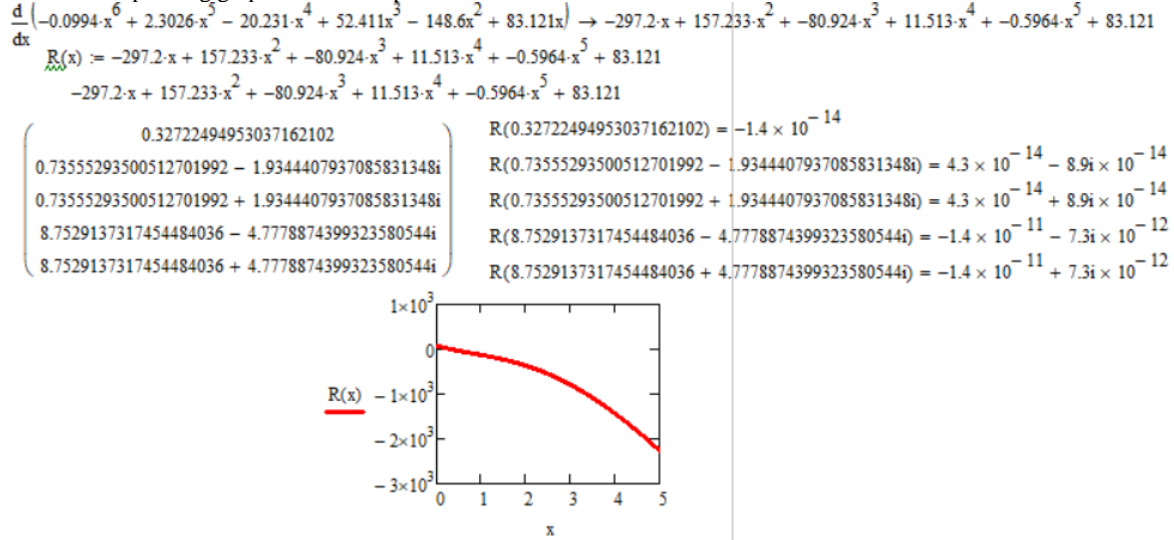


**FIGURE 9**. Algorithmic process of finding extreme values of $D(v_2)$

To assess the significance of the obtained result, the spectral analysis of the analytical functions used in the form of the amplitude-frequency characteristics of these polynomials, performed by the fast Fourier transform method, was carried out, figure 9.
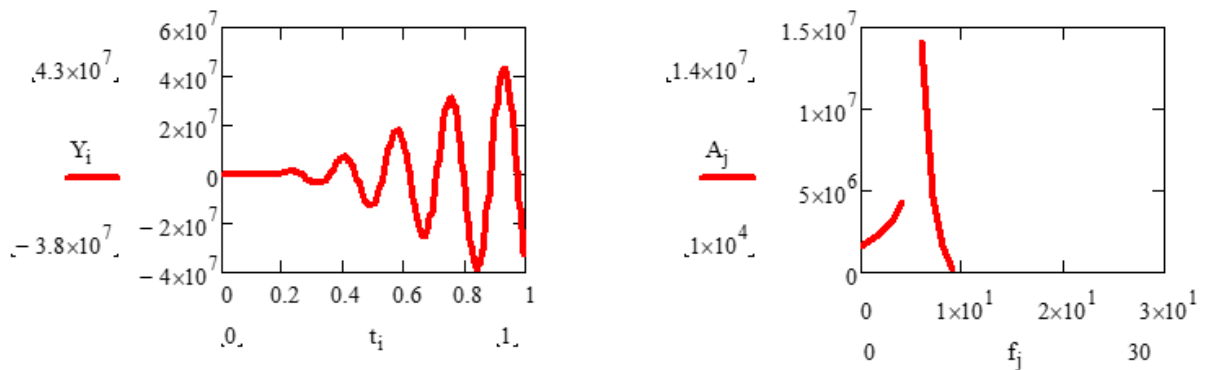


**FIGURE 10**. Spectral analysis of the regression model

The obtained regression model of the two-parameter function $D(v_1, v_2)$ indicates that in the given low-frequency frequency range, up to 6 Hz, there is only one spectral frequency of the order of 100 Hz, which tends to decrease in amplitude and does not exceed the resonant threshold amplitude values of L. Euler's critical parameters accepted in the work.

## CONCLUSIONS

The answer to the question "is the structure stable or unstable, or a separate element of this structure?" is a very important task. As it is shown in this paper, even a minor cause is sufficient for the loss of stability of a structure that has reached a critical state. If the process of loss of stability has "already" begun, then it goes very quickly and leads to a sharp change in the original shape, and very often, in addition to this, to the destruction of parts or the entire structure. The values of critical forces, and the relationship between the various parameters of the structure we are analyzing in its

different states, found in this work, indicate that with the help of the method proposed in this work, it is fundamentally possible to predict the behavior of the structure being developed at the stage of its calculation and design. The methodology proposed in this paper allows you to be confident in the results of calculations and in the results of design of the structure at all stages of its life cycle.

## Results

a) The method of forecasting the loss of stability of the analyzed structure is presented;
b) The problem of stability of the form of the analyzed system is solved;
c) For the analyzed structure, possible destructive factors have been identified in the form of critical impacts that potentially have the ability to disable it, (Fig. 7,);
d) Analytical expressions are found between the critical parameters and the equilibrium conditions of the system (Fig. 8);
e) The extreme values of the function D(v1,v2) in the considered intervals are found Fig. 8,9;
f) The amplitude-frequency analysis of the analytical models of the stability equation (10) of the analyzed system found in the work is performed Fig. 10.

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