

# SVM Assisted Primary User-Detection for Non-Cooperative Cognitive Radio Networks

Kais Bouallegue, Matthieu Crussière and Sofiane Kharbech

EasyChair preprints are intended for rapid dissemination of research results and are integrated with the rest of EasyChair.

July 19, 2020

# SVM Assisted Primary User-Detection for Non-Cooperative Cognitive Radio Networks

Kais Bouallegue Matthieu Crussiere Univ Rennes, INSA Rennes Univ Rennes, INSA Rennes CNRS, IETR - UMR 6164 CNRS, IETR - UMR 6164 Rennes, France Rennes, France kais.bouallegue@insa-rennes.fr matthieu.crussiere@insa-rennes.fr

Sofiane Kharbech University of Lille, IEMN/IRCICA, Lille, France Tunis El Manar University, Sys'Com-ENIT, Tunis, Tunisia sofiane.kharbech@ieee.org

Abstract—This paper presents a new blind spectrum sensing (SS) algorithm based on a machine learning model: the radial basis function support-vector machines (RBF-SVM). As features, the introduced approach uses statistical tests that are based on the eigenvalues of the received signals covariance matrix. Since the decision on the frequency resource occupancy is in fact an issue of labeling binary data, SVM is intended as a potential technique for SS paradigm. The flexibility of SVM for linearly non-separable and high dimensional data makes it a good candidate for our issue, particularly that we consider low signal to noise ratios (SNR). Computer simulations shows that the proposal outperforms classical non-cooperative SS algorithms.

Index Terms-Cognitive radio, spectrum sensing, supportvector machines, eigenvalue decomposition.

# I. INTRODUCTION

For several years now, the demand in terms of spectral resources and its under-exploitation are increasing [1]. This issue is all the more relevant today when many new technologies, such as the Internet of things (IoT) [2] or vehicle to everything communication (V2X) [3], accelerate the saturation of the spectrum. This context urges to find new reliable solutions for future requests within the frequency resources. It is in these circumstances that cognitive radio (CR) could be the most well-grounded alternative. Indeed, by allowing some users, called secondary user (SU), to exploit the unused frequency resources opportunistically, the spectrum occupancy will be higher and finally better optimized. However, the licensed user, called primary user (PU), must not be subject to interference from the CR communication network. The backbone of the CR communication is to find the best resource opportunity in order to dynamically reallocate free frequency band or white spaces, as depicted in Fig 1. This mechanism must happen in the best possible and most reliable way. Thus, the SU has to be aware of its environment to avoid dismissing spectral opportunities but also to not interfere with PU communication.

The spearhead of CR systems is the cognition loop that is built upon the spectrum sensing (SS) algorithms [4]. A standard for wireless regional area networks (WRANs) with a CR-based air interface is in progress by the IEEE 802.22 Working Group. This standard envisages to wield very-high frequency (VHF), and ultra-high frequency (UHF) bands currently licensed for analog and digital television broadcasting (between 54 MHz to 862 MHz) and wireless microphones and

potentially in the 1300 MHz to 1750 MHz, and 2700 MHz to 3700 MHz provided the regulatory regime allows it [5]. The CR system requires to sense the licensed user at a probability of detection of at least 90% and a probability of false alarm lower than 10% under shallow SNR region as illustrated in Table I.

SS paradigm has sparked a major interest these last years and resulted in an extensive literature proposing several algorithms [6]. Many SS algorithms have been developed, such as cyclostationarity detector (CD) [7], energy detection (ED) [8] or matched-filter (MF) [9]. These algorithms consider that the SU has a priori knowledge of the source signal, the noise power, or the channel.

In order to improve the detection of the PU, a set of blind SS algorithms that exploit the multiple-input multiple-output (MIMO) feature are investigated: arithmetic-to-geometric mean (AGM) [10], maximum-eigenvalue-geometric-mean (MEGM) [11], mean-to-square extreme eigenvalue (MSEE) [12], blindly combined energy detection (BCED) [13], and unified sensing algorithm (USA) [14].

To enhance the performance of the PU detection [15], recent contributions have proposed the spatial filtering such as maximum-to-minimum beam energy (MMBE) [16], maximum-to-mean energy detector (MMED) [17], and maximum energy beamforming-output-to-input (MEBOI) [18]. But these algorithms, based on beamforming, are under ray propagation assumption. Other contributions for SS algorithms are based on the deep learning approach [19] or machine learning (ML) [20]. These two last methods provide good performance gains for cooperative sensing methods. Under the non-cooperative assumption, the ML aspect was not explored enough.

In this paper, we propose a new fully blind method in a non-cooperative context. Based on the SVM approach, this method uses three statistical tests, based on eigenvalues of the covariance matrix of the received signal, as input features. Unlike conventional thresholding methods, the non-linear samples separation, as well as the maximization of the margin, improve detection performance. Thus it allows better separability for the two subspaces  $(\mathcal{H}_1/\mathcal{H}_0)$ .

The rest of the paper is organized as follows. In Section II, we describe the system model and the background work,

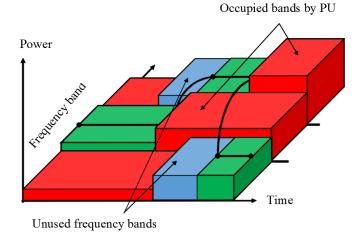


Fig. 1. Scenario of dynamic resource allocation for secondary user over free frequency in narrowband context.

including related work. Section III describes the contribution of the paper through the ML approach, notably, the radial basis function SVM (RBF-SVM). Section IV provides computerbased performance analysis. Conclusions are drawn in Section V.

Boldface lower letter is used to denote vectors and boldface capital letter to denote matrices. We use superscript  $(.)^T$  to denote transpose.  $\mathbf{I}_q$  denotes the identity matrix of order q and E[.] stands for expectation operation.

#### **II. SYSTEM MODEL**

Here, we assume a SU as a CR system with M (M > 1) linear antennas. In SS model, there is two main hypothesis  $\mathcal{H}_1$ , when the PU is present and  $\mathcal{H}_0$ , when the frequency resource is vacant. Thus, set of probability is defined

- False alarm noted  $P_{fa} = \Pr(\mathcal{H}_1 \mid \mathcal{H}_0)$
- Miss-detection noted  $P_{md} = \Pr(\mathcal{H}_0 \mid \mathcal{H}_1)$
- Detection noted  $P_d = 1 P_{md} = \Pr(\mathcal{H}_1 \mid \mathcal{H}_1).$

To provide clarity, in Fig 2, we illustrate an example of the probability density function of the two hypothesis ( $\mathcal{H}_1$  and  $\mathcal{H}_0$  in continuous line and in dashed line curve respectively). The threshold makes it possible to distinguish between the  $P_{md}$  (red area) and the  $P_{fa}$  (green area). Some standards of SS are depicted in Table I. For example, for wireless microphone, the requirement of this standard is to identify the PU at a SNR of at least -12dB with a probability of detection over 0.9 [21].

The received signal by the SU is expressed as

$$y_m(n) = \alpha r_m(n) + g_m(n), \quad n = 1, 2, \cdots, N$$
 (1)

where N is the number of observed samples, m denotes the antenna  $(m = 0, \dots, M - 1)$ , y is the received signal, r(n) represents the transmitted signal from the PU, g(n) is a zeromean additive white Gaussian noise with variance  $\sigma_b^2$  and  $\alpha =$ 

TABLE IFEATURES OF STANDARD [5]

Features	Analog TV	Digital TV	Wireless Microphone
Probability of detection	90%	90%	90%
Probability of false alarm	10%	10%	10%
Time of detection (second)	$\leq 2$	$\leq 2$	$\leq 2$
SNR (dB)	1	-21	-12

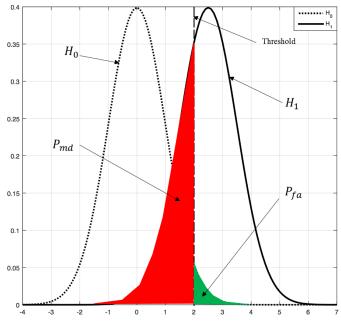


Fig. 2. Example of probability density function of  $\mathcal{H}_1$  and  $\mathcal{H}_0$  and representation of the probability of error

 $\{0,1\}$  under  $\mathcal{H}_{\alpha}$  hypothesis. The vector representation of the received signal under  $\mathcal{H}_1$  hypothesis is written

$$\mathbf{y}(n) = \sum_{p=1}^{P} \sum_{k=0}^{C_p} \mathbf{h}_p(k) s_p(n-k) + \mathbf{g}(n),$$
(2)

where P is the number of PU,  $C_p$  is the channel order,  $\mathbf{h}_p = [h_{1p}, h_{2p}, \cdots, h_{Mp}]^T$  is the channel from PU<sub>p</sub> to receiver antennas. We consider L consecutive samples and define the following vectors

$$\mathbf{y}_L(n) = [\mathbf{y}^T(n), \cdots, \mathbf{y}^T(n-L+1)]^T, \qquad (3)$$

$$\mathbf{g}_L(n) = [\mathbf{g}^T(n), \cdots, \mathbf{g}^T(n-L+1)]^T,$$
(4)

$$\mathbf{s}_L(n) = [\mathbf{s}_0^T(n), \mathbf{s}_1^T(n), \cdots, \mathbf{s}_{P-1}^T(n)]^T,$$
(5)

where  $\mathbf{s}_p^T(n) = [s_p(n), s_p(n-1), \cdots, s_p(n-L-C_p+1)]^T$ . *L* is called the smoothing factor and is well investigated in [15]. The matrix expression of the received signal is given by

$$\mathbf{y}_L(n) = \mathbf{H}\mathbf{s}_L(n) + \mathbf{g}_L(n), \tag{6}$$

where **H** is a  $ML \times (C + PL)$  matrix which represents the channel  $C = \sum_{p=1}^{P} C_p$ . The theoretical covariance matrix of the received signal is expressed as

$$\mathbf{R}_y = \mathbf{H}\mathbf{R}_s\mathbf{H}^H + \sigma_g^2\mathbf{I}_{ML},\tag{7}$$

where

 $\mathbf{R}_{y}$ 

$$= E[\mathbf{y}_L(n)\mathbf{y}_L(n)^H]$$
(8)

$$\mathbf{R}_s = E[\mathbf{s}_L(n)\mathbf{s}_L(n)^H].$$
(9)

The eigenvalues, noted  $\lambda_i$ , of the covariance matrix (Hermitian matrix) of the received signal are real numbers. The ML eigenvalues are defined as  $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_{ML}$ . The estimated covariance matrix is given by

$$\hat{\mathbf{R}}_{y}(N) = \frac{1}{N} \sum_{k=1}^{N} \mathbf{y}_{L}(k) \mathbf{y}_{L}(k)^{H}.$$
(10)

The noise subspace size is equal to (M - P)L - C, so we can note the estimated eigenvalues as  $\hat{\lambda}_1 > \hat{\lambda}_2 > \cdots > \hat{\lambda}_{C+PL} > \hat{\lambda}_{C+PL+1} = \cdots = \hat{\lambda}_{ML} = \sigma_g^2$ . Some of SS algorithms exploit these estimated eigenvalues. The following paragraphs provide three well-known algorithms of eigenvalue based non-cooperative SS.

a) Arithmetic to Geometric Mean (AGM): This SS algorithm is based on the ratio between the mean received energy and geometric mean of the eigenvalue [10]. The statistical test expressed as

$$T_{\text{AGM}} = \frac{\frac{1}{ML} \sum_{k=1}^{ML} \hat{\lambda}_k}{\left(\prod_{k=1}^{ML} \hat{\lambda}_k\right)^{1/ML}} \stackrel{\mathcal{H}_0}{\underset{\mathcal{H}_1}{\lessgtr}} \gamma_{\text{AGM}},\tag{11}$$

where  $\gamma_{AGM}$  is the threshold [11] given by

$$\gamma_{\text{AGM}} = \sqrt{\frac{2}{MLN}} Q^{-1} \left( P_{fa} \right) + 1. \tag{12}$$

where  $Q^{-1}$  is the inverse Q-function which describes the tail distribution function of the standard normal distribution

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int_{t}^{+\infty} \exp(-u^2/2) \,\mathrm{d}u.$$
 (13)

b) Blindly Combined Energy Detection (BCED): In [13], the statistical test is

$$T_{\text{BCED}} = \frac{\hat{\lambda}_1}{\frac{1}{ML} \sum_{k=1}^{ML} \hat{\lambda}_k} \stackrel{\mathcal{H}_0}{\underset{\mathcal{H}_1}{\leq}} \gamma_{\text{BCED}}, \tag{14}$$

where  $\gamma_{\text{BCED}}$  is the threshold developed in [22]. Considering the ratio between the largest eigenvalue and the mean estimated energy, the probability of false alarm is given by

$$P_{fa} = 1 - F_{\text{TW}} \left( \frac{\gamma_{\text{BCED}} - \mu_{N,ML}}{\sigma_{N,ML}} \right) + \frac{1}{MLN} \left( \frac{\mu_{N,ML}}{\sigma_{N,ML}} \right)^2 F_{\text{TW}}'' \left( \frac{\gamma_{\text{BCED}} - \mu_{N,ML}}{\sigma_{N,ML}} \right), \quad (15)$$

where  $F_{\text{TW}}$  is the cumulative distribution function of the Tracy-Widom (TW) distribution and  $F_{TW}''$  represents the second derivate of  $F_{\rm TW}$ ,

$$\mu_{N,ML} = \left(1 + \sqrt{\frac{ML}{N}}\right)^2 \tag{16}$$

$$\sigma_{N,ML} = N^{\frac{-2}{3}} \left( 1 + \sqrt{\frac{ML}{N}} \right) \left( 1 + \frac{1}{\sqrt{\frac{ML}{N}}} \right)^{1/3}.$$
 (17)

# c) Maximum-Eigenvalue-to-the-Geometric-Mean

(MEGM): The statistical test of MEGM algorithm is based on the the ratio between the largest eigenvalue and the geometric mean of the eigenvalues [11]

$$T_{\text{MEGM}} = \frac{\lambda_1}{\left(\prod_{k=1}^{ML} \lambda_k\right)^{1/ML}} \stackrel{\mathcal{H}_0}{\underset{\mathcal{H}_1}{\lesssim}} \gamma_{\text{MEGM}}.$$
 (18)

The threshold is defined as

w

$$\gamma_{\text{MEGM}} = \frac{F_{\text{TW}}^{-1} \left(1 - P_{fa}\right) \nu + \chi}{N},$$
(19)

where 
$$\nu = \sqrt{\chi} \left( \frac{1}{\sqrt{N-1}} + \frac{1}{\sqrt{ML}} \right)^{(1/3)}$$
 and  $\chi = \left( \sqrt{N-1} + \sqrt{ML} \right)^2$ .

These algorithms cited above provide best performance in non-cooperative spectrum sensing context.

#### III. SVM-BASED SPECTRUM SENSING (SVM-SS)

In this section, an SS scheme with a statistical test based on the eigenvalue decomposition of the covariance matrix of the received signal and radial basis function-support vector machine (RBF-SVM) is established. SVM is a supervised machine learning algorithm exploited to solve discrimination problems. SVM is designed for binary classification, which makes it a good candidate for the SS paradigm ( $\mathcal{H}_0$  or  $\mathcal{H}_1$ ).

SVM aims at finding the best hyperplane in order to separate between a set of input features. Thus, this decision plane is designed from  $j(j = 1, \dots, \ell)$  points training dataset of pairs  $(t_i, f_j)$ , where  $t_j$  is the input parameter vector and  $f_j$  in  $(1,-1)^{j}$  indicates the class to which the vector  $t_{j}$  belongs. The whole hyperplane is expressed as the set of points t satisfying  $w^T t + b = 0$  and the best decision plane is designed by solving the following optimization parameter

$$\min_{\boldsymbol{w},\boldsymbol{b},\boldsymbol{\xi}} \qquad \frac{1}{2}\boldsymbol{w}^T\boldsymbol{w} + V\sum_{i=1}^{\ell}\xi_i \qquad (20)$$

subject to 
$$f_j(\boldsymbol{w}^T \varphi(\boldsymbol{t}_j) + b) \ge 1 - \xi_j,$$
 (21)

where  $\xi_i$  is a slack variable, V > 0 is a penalty parameter and  $\varphi(.)$  is the function which maps each training data point  $t_i$  to a high-dimensional space using a kernel function K(.)which satisfies  $K(t_j, t_i) = \varphi(t_j)^T \varphi(t_i)$ . SVM algorithms use different kind of kernel function. The most exploited type of

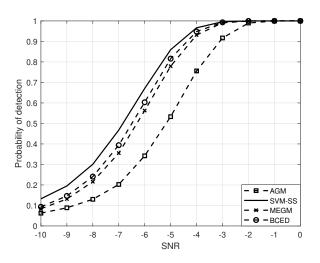


Fig. 3. Probability of detection versus SNR

kernel function is the radial basis function (RBF) expressed as

$$\boldsymbol{K}(t_j, t_i) = \exp\left(-\varrho \mid \mid t_j - t_i \mid \mid^2\right).$$
(22)

In this paper, the Gaussian kernel, is used  $(\varrho = \frac{1}{2\sigma})$  where  $\sigma$  is the width of the kernel function. The SVM model, i.e., penalty parameter *C* and  $\sigma$ , is validated using the well-known cross-validation technique. Here, the inputs of the SVM are the statistical tests based on the eigenvalues. The main advantages of the SVM are the good trade-off between the lower number of training samples and the accuracy, in addition to the faster classification compared to the other ML algorithms [23].

# **IV. SIMULATION RESULTS**

In this section, we furnish some simulation results comparing the proposed method, SVM-SS, to the other ones based on the eigenvalue decomposition of the covariance matrix of the received signal for different scenarios. Computer simulations are based on  $10^5$  Monte-Carlo trials. Unless otherwise indicated, we assume throughout simulations the same values for the following parameters P = 1, M = 6, N = 20,  $P_{fa} = 0.1$ and  $C_p = 0$ . The training set is set to 120 samples for each hypothesis  $\mathcal{H}_1$  and  $\mathcal{H}_0$ .

In Fig. 3, we illustrate the performance of the SVM-SS method comparing to the other ones in term of  $P_d$  versus the SNR. The SVM-SS method offers better performance, especially for low SNR region (-10 dB to -5 dB). We can note that our proposal provides more than 5% of better detection relatively to the other methods.

In Fig. 4, we evaluate the performance of the SVM-SS method with less number of antennas (M = 4). As expected, the detection performance degrades when comparing to Fig. 3, but the gap of  $P_d$  is also the same, 5% of better detection for SVM-SS method. We can note that the SVM-SS is not sensitive to the number of antennas. This makes it possible to exploit this method, whatever the spatial diversity of the system.

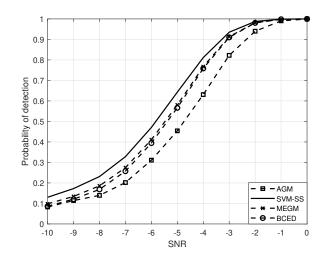


Fig. 4. Probability of detection versus SNR for M = 4

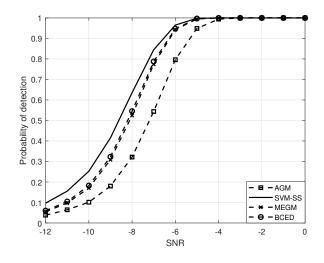


Fig. 5. Probability of detection versus SNR for N = 40

Fig. 5 evaluates the impact of the number of samples by setting a signal duration required for sensing of N = 40. Analyzing Figures 3 to 5, we observe an improvement in the detection rate, with a difference of about 5%. It still clearly appears that the SVM-SS algorithm outperforms other methods, although the difference with other methods slightly reduces. This means that the SVM-SS approach is all the more interesting than the sensing duration decreases, which makes it an efficient sensing method.

Finally, the influence of the number of paths  $C_p$  on the performance can be analyzed from Fig. 6 where  $C_p$  is set to 2 whereas other results were obtained in a pure line-of-sight channel setting (Fig. 3 to 5). This allows us to compare the proposed algorithm to those based on the eigenvalues considering different frequency selectivity rates. As can be expected, the overall performance of the various methods degrade when increasing the number of paths. However, when the frequency selectivity is high ( $C_p = 2$ ), the performance

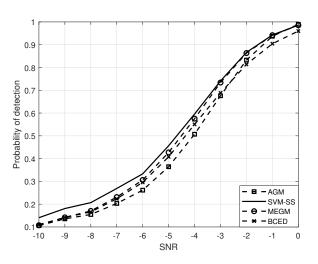


Fig. 6. Probability of detection versus SNR for  $C_p = 2$ 

of the SVM-SS still outperforms other methods. We hence conclude that even if the proposed method is sensitive to the number of paths as others, it still keeps better performance.

# V. CONCLUSION

In this paper, we proposed a new blind method in a noncooperative context. Considering the RBF-SVM approach, this method exploits three techniques (BCED, MEGM, and AGM) based on the eigenvalues of the received signals covariance matrix. In opposition to classical algorithms of spectrum sensing, which are based on the fixed threshold, our method proposes a non-linear separation based on training samples. From various simulation trials, it turns out that the proposed SVM-SS approach outperforms other methods built on eigenvalue decomposition, whatever various parameter settings such as channel selectivity, number of antennas and sensing duration.

# REFERENCES

- F. C. Comm. Facilitating Opportunities for Flexible, Efficient, and Reliable Spectrum Use Employing Cognitive Radio Technologies, Feb. 25, 2010.
- [2] H. Kim, S. Ko, and S. Kim, "Cognitive random access for internet-ofthings networks," in 2017 IEEE 85th Vehicular Technology Conference (VTC Spring), pp. 1–5, June 2017.
- [3] K. V. Mishra, A. Zhitnikov, and Y. C. Eldar, "Spectrum sharing solution for automotive radar," in 2017 IEEE 85th Vehicular Technology Conference (VTC Spring), pp. 1–5, June 2017.
- [4] J. Mitola and J. Maguire, G.Q., "Cognitive radio: making software radios more personal," *IEEE Personal Communications*, vol. 6, pp. 13–18, Aug. 1999.
- [5] "802.22-2019 IEEE Approved Draft Standard for Information Technology Local and Metropolitan Area Networks Specific Requirements Part 22: Cognitive Radio Wireless Regional Area Networks (WRAN) Medium Access Control (MAC) and Physical Layer (PHY) Specifications: Policies and Procedures for Operation in the Bands that Allow Spectrum Sharing where the Communications Devices may Opportunistically Operate in the Spectrum of the Primary Service," IEEE Computer Society, 2019.
- [6] A. Ali and W. Hamouda, "Advances on spectrum sensing for cognitive radio networks: Theory and applications," *IEEE Communications Surveys Tutorials*, vol. 19, pp. 1277–1304, Secondquarter 2017.

- [7] W. M. Jang, "Blind cyclostationary spectrum sensing in cognitive radios," *IEEE Communications Letters*, vol. 18, pp. 393–396, March 2014.
- [8] L. Tlebaldiyeva, T. A. Tsiftsis, and B. Maham, "Performance analysis of improved energy detector with hardware impairments for accurate spectrum sensing," *IEEE Access*, vol. 7, pp. 13927–13938, 2019.
- [9] A. A. Kabeel, A. H. Hussein, A. A. Khalaf, and H. F. Hamed, "A utilization of multiple antenna elements for matched filter based spectrum sensing performance enhancement in cognitive radio system," *AEU International Journal of Electronics and Communications*, vol. 107, pp. 98 109, 2019.
- [10] R. Zhang, T. J. Lim, Y.-C. Liang, and Y. Zeng, "Multi-antenna based spectrum sensing for cognitive radios: A glrt approach," *Communications, IEEE Transactions on*, vol. 58, pp. 84–88, January 2010.
- [11] N. Pillay and H. J. Xu, "Blind eigenvalue-based spectrum sensing for cognitive radio networks," *IET Communications*, vol. 6, pp. 1388–1396, July 2012.
- [12] K. Bouallegue, I. Dayoub, M. Gharbi, and K. Hassan, "Blind spectrum sensing using extreme eigenvalues for cognitive radio networks," *IEEE Communications Letters*, vol. 22, pp. 1386–1389, July 2018.
- [13] Y. Zeng, Y.-C. Liang, and R. Zhang, "Blindly combined energy detection for spectrum sensing in cognitive radio," *IEEE Signal Processing Letters*, vol. 15, pp. 649–652, 2008.
- [14] K. Bouallegue, J. Baudais, and M. Crussière, "Unified sensing algorithm : a smart full exploitation of detection methods," in 2019 International Conference on Wireless and Mobile Computing, Networking and Communications (WiMob), pp. 1–6, Oct 2019.
- [15] K. Bouallegue, M. Crussière, and I. Dayoub, "On the impact of the covariance matrix size for spectrum sensing methods : beamforming versus eigenvalues," in *IEEE ISCC 2019 - 2019 IEEE Symposium on Computers and Communications*, July 2019.
- [16] K. Bouallegue, I. Dayoub, M. Gharbi, and K. Hassan, "A cost-effective approach for spectrum sensing using beamforming," *Physical Communication*, vol. 22, no. Supplement C, pp. 1 – 8, 2017.
- [17] K. Bouallegue, I. Dayoub, and M. Gharbi, "Spectrum sensing for wireless communications using energy ratio and beamforming," in 2017 IEEE International Conference on Communications (ICC), pp. 1–6, May 2017.
- [18] K. Bouallegue, I. Dayoub, and M. Gharbi, "On low-complexity spectrum sensing: Analytical approach based spatial scanning," in 2017 IEEE Wireless Communications and Networking Conference (WCNC), pp. 1– 6, March 2017.
- [19] J. Gao, X. Yi, C. Zhong, X. Chen, and Z. Zhang, "Deep learning for spectrum sensing," *IEEE Wireless Communications Letters*, vol. 8, pp. 1727–1730, Dec 2019.
- [20] K. M. Thilina, K. W. Choi, N. Saquib, and E. Hossain, "Machine learning techniques for cooperative spectrum sensing in cognitive radio networks," *IEEE Journal on Selected Areas in Communications*, vol. 31, pp. 2209–2221, November 2013.
- [21] M. Naraghi-Pour and T. Ikuma, "Autocorrelation-based spectrum sensing for cognitive radios," *Vehicular Technology, IEEE Transactions on*, vol. 59, pp. 718–733, Feb 2010.
- [22] I. M. Johnstone, "On the distribution of the largest eigenvalue in principal components analysis," *The Annals of Statistics*, vol. 29, pp. 295–327, 04 2001.
- [23] I. Ahmad, M. Basheri, M. J. Iqbal, and A. Rahim, "Performance comparison of support vector machine, random forest, and extreme learning machine for intrusion detection," *IEEE Access*, vol. 6, pp. 33789–33795, 2018.