Investigation of Rotation on Plane Waves Under Two-Temperature Theory in Generalized Thermoelasticity

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October 10, 2021

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Abstract: The present paper is aimed at a detailed analysis of the effects of rotation on the propagation of harmonic plane waves under two-temperature thermoelasticity theory. We consider a homogeneous and isotropic elastic medium that is rotating with uniform angular velocity. After formulating the problem, we obtain the dispersion relations for the longitudinal and transverse plane waves propagating in the medium and the solutions of dispersion relations are obtained analytically. The asymptotic expressions of several important characterizations of the wave fields are obtained for high frequency as well as for low frequency values. In order to observe the behavior of the wave characterizations for the intermediate values of frequency and to examine the effects of rotation on them, computational work is carried out to find the numerical values of different wave fields for intermediate values of frequency and for various angle of rotation. The results are shown graphically. An in-depth analysis of the effects of rotation on plane wave is presented on the basis of our analytical and numerical results.

Keywords: Two-temperature thermoelasticity; Generalized thermoelasticity; Harmonic plane wave; Rotating elastic body; Centripetal and Coriolis acceleration.

1. **Introduction**:

In the present work we propose to investigate the propagation of harmonic plane waves in an infinite rotating elastic medium under two-temperature thermoelasticity theory. The propagation of harmonic plane waves in elastic medium have been the subject of interest during several years. Chadwick and Sneddon [4] and Chadwick [5] studied the propagation of plane waves in classical thermoelasticity. The propagation of plane waves in the context of generalized thermoelasticity with one relaxation time introduced by Lord and Shulamn [21] is discussed by Nayfeh and Nemat-Nasser [23] and later on by Puri [29]. The propagation and stability of harmonically time-dependent thermoelastic plane waves in temperature-rate-dependent thermoelasticity theory developed by Green and Lindsay [15] is reported by Agarwal [1]. Investigation on plane waves in the context of thermoelasticity theory without energy dissipation (Green–Naghdi [17]) is discussed by Chandrasekharaiah [10]. In a recent work Puri and Jordan [26] have investigated the propagation of plane waves in the context of GN-III thermoelasticity theory [16]. Wave propagation in infinite rotating elastic solid medium was investigated by Schenberg and Censor[34] and later on by several other researchers like Puri [28], Chandrasekharaiah and Srikantiah [7], Roychoudhuri [33], Roychoudhuri and Mukhopadhyay[32], Roychoudhuri and Bandyopadhyay [31], Chandrasekharaiah [8, 9], Othman[24], Auriault[3], Sharma and Othman[35]. [11,12] formulated the two-temperature thermoelasticity theory and this theory proposes that the heat conduction on a deformable body depends on two different temperatures: the conductive temperature, φ and the thermodynamic temperature, θ. This theory suggests that the difference between these two temperatures is proportional to the heat supply and in absence of heat supply the two temperatures are equal for time-independent situation [11]. However, for time dependent cases the two temperatures are in general different, regardless of the heat supply. Uniqueness and reciprocity theorems for the two-temperature thermoelasticity theory in case of a homogeneous and isotropic solid have been provided by Iesan [18]. Subsequently, several investigations (see Warren and Chen [37], Warren [38], Amos[2], Chakrabarti [6], Ting[36], Colton and Wimp [14]) have been pursued by employing the linearized version of this theory. This two-temperature thermoelasticity theory has again aroused much interest in the recent years. The existence, structural stability, convergence and spatial behavior of two-temperature thermoelasticity theory have been discussed in details by Quintanilla [30]. The propagation of harmonic plane waves in the same theory is discussed by Puri and Jordan [27]. Several other research works [39- 42, 20, 22, 19] have also been carried out very recently on the basis of this two-temperature thermoelasticity theory indicated some significant features of this theory.

The main objective of the present study is to investigate the effects of rotation on the propagation of plane harmonic waves in a homogeneous and isotropic rotating elastic medium in the context of the linear theory of two-temperature thermoelasticity. After obtaining the dispersion relation solutions of both the longitudinal and transverse plane waves, we find the asymptotic expansions of several qualitative characterizations of the wave fields such as phase velocity, specific loss, penetration depth, amplitude coefficient factor and phase shift of the thermodynamic temperature for the high and low frequency values. It should be mentioned here that in earlier studies concerning plane waves only the behavior of the phase velocity, specific loss, penetration depth etc. are discussed on the basis of the asymptotic expressions for the high and low frequency values. However, being motivated by the work reported by Puri and Jordan [27] we also make an attempt to observe the behavior of the above mentioned quantities for intermediate values of frequency with the help of computational work. For this the numerical values of the wave characterizations for intermediate values of frequency and for various values of rotational angle are computed and the results are plotted in several graphs. A detailed analysis of the numerical results highlighting the effects of rotation on various wave fields is presented and the analytical results are verified with the help of our numerical results. The results are also compared with the corresponding results of the case of absence of rotation as reported by Puri and Jordan [27].

2. Basic governing equations

We consider a linear homogeneous isotropic thermally conducting elastic medium rotating uniformly with the angular velocity $\boldsymbol{\Omega} = \Omega_0\, \boldsymbol{p}$, where $\boldsymbol{p}$ is the unit vector that represents the direction of the axis of rotation. The displacement equation of motion in the rotating frame of reference involves two additional terms- the centripetal acceleration $\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{u})$ due to the time-varying motion only and the Co-riolis acceleration $2\boldsymbol{\Omega} \times \dot{\boldsymbol{u}}$, where $\boldsymbol{u}$ is the displacement vector. The equations governing the displacement and thermal fields in the absence of body forces and heat sources under two temperature thermoelasticity theory are therefore taken in usual notations as follows:

Stress - strain temperature relations:

$$\sigma_{ij} = \lambda e \delta_{ij} + 2\mu e_{ij} - \gamma \theta \delta_{ij} \tag{1}$$

Strain - displacement relations: $$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{2}$$

Heat conduction equation without heat source:

$$K\varphi_{,ii} = \rho c_E \frac{\partial \theta}{\partial t} + \gamma \Phi_0 \frac{\partial e}{\partial t} \tag{3}$$

where e is the dilatation and defined as $e = e_{kk}$

The stress equation of motion in a rotating medium without body force:

$$\sigma_{ji,j} = \rho\left[\ddot{\boldsymbol{u}}_i + \left[\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{u})\right]_i + (2\boldsymbol{\Omega}\times\dot{\boldsymbol{u}})_i\right] \tag{4}$$

The conductive temperature φ is related to the thermodynamic temperature θ as

$$\varphi - \theta = \alpha\varphi_{,ii} \tag{5}$$

where α >0 (a scalar) is the two-temperature parameter.

3. Problem formulation

Using Eqs. (1), (2) and (5) we write the equation of motion (4) in the form

$$(\lambda+\mu)\nabla(\nabla\bullet\boldsymbol{u}_i) + \mu\nabla^2\boldsymbol{u}_i - \gamma\,\nabla(\varphi - \alpha\varphi_{,ii}) = \rho\left[\ddot{\boldsymbol{u}}_i + (\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{u}))_i + (2\boldsymbol{\Omega}\times\dot{\boldsymbol{u}})_i\right] \tag{6}$$

From Eqs. (3) and (5) we get

$$\left[K + \alpha\rho c_E\frac{\partial}{\partial t}\right]\nabla^2\varphi = \rho c_E\dot{\varphi} + \gamma\Phi_0\nabla\bullet\dot{u}_i \tag{7}$$

We introduce the following dimensionless quantities and notations:

$$x_i' = c_0\eta x_i,\quad t' = c_0\eta^2 t,\ \varphi' = \frac{\varphi}{\varphi_0},\ u_i' = c_0\eta u_i,\ c_0{}^2 = \frac{(\lambda+2\mu)}{\rho},\ \ \eta = \frac{\rho c_E}{K},\ \Omega_0{}' = \frac{\Omega_0}{c_0^2\eta}$$

$$a_1 = \frac{\gamma\varphi_0}{(\lambda+2\mu)},\quad a_2 = \frac{\gamma}{K\eta},\ \alpha' = c_0^2\eta^2\alpha,\ \mu_1 = \frac{\mu}{\lambda+2\mu},\ \lambda_1 = 1-\mu_1$$

Equation (6) and (7) then transform to the dimensionless forms (after dropping the primes for convenience) as

$$\lambda_1\nabla(\nabla\bullet\boldsymbol{u}_i) + \mu_1(\nabla^2\boldsymbol{u}_i) - a_1\nabla(\varphi - \alpha\varphi_{,ii}) = \ddot{\boldsymbol{u}}_i + (\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{u}))_i + (2\boldsymbol{\Omega}\times\dot{\boldsymbol{u}})_i \tag{8}$$

$$\left[1 + \alpha\frac{\partial}{\partial t}\right]\nabla^2\varphi = \frac{\partial\varphi}{\partial t} + a_2\nabla\bullet\dot{\boldsymbol{u}}_i \tag{9}$$

4. Plane harmonic waves solutions

To study the propagation of plane harmonic waves the solutions of equation (8) and (9) are assumed in the form

$$(\boldsymbol{u},\varphi) = (\boldsymbol{a},b)\exp\left[i(\omega t - \eta\boldsymbol{n}\bullet x)\right] \tag{10}$$

where $\boldsymbol{a}$ is a vector and b is an arbitrary constant not both zero. ω is the frequency, η is the wave number of the wave. $\boldsymbol{n}$ is the unit vector along the direction of propagation. ω is assumed to be a positive real and $\boldsymbol{a}, b, \eta$ are allowed to be complex.

Substituting these expansions into the equations (8) and (9) we get

$$\left(\omega^2 + \Omega_0{}^2 - \mu_1\eta^2\right)\boldsymbol{a} - \lambda_1\eta^2\left(\boldsymbol{a}\bullet\boldsymbol{n}\right)\boldsymbol{n} - \left(\boldsymbol{\Omega}\bullet\boldsymbol{a}\right)\boldsymbol{\Omega} + 2i\omega\left(\boldsymbol{\Omega}\times\boldsymbol{a}\right) = ia_1 b\eta\left(1+\alpha\eta^2\right)\boldsymbol{n} \tag{11}$$

$$\left[-\eta^2 - i\omega\left(1+\alpha\eta^2\right)\right]b = a_2\left(\boldsymbol{a}\bullet\boldsymbol{n}\right)\eta\omega \tag{12}$$

We note that if $\boldsymbol{a} = 0$, then equation (11) yields $b = 0$, but $\boldsymbol{a}$ and b cannot vanish for the waves of the desired type to occur. Therefore, we take $\boldsymbol{a}$ to be a nonzero vector. We analyze purely shear waves and purely dilatational waves on the basis of equations (11) and (12).

Case I: Shear wave

For purely shear waves, we have $\boldsymbol{a}\bullet\boldsymbol{n} = 0$, and equations (11) and (12) therefore become

$$\left(\omega^2 + \Omega^2 - \mu_1\eta^2\right)\boldsymbol{a} - \left(\boldsymbol{\Omega}\bullet\boldsymbol{a}\right)\boldsymbol{\Omega} + 2i\omega\left(\boldsymbol{\Omega}\times\boldsymbol{a}\right) = ia_1 b\eta\left(1+\alpha\eta^2\right)\boldsymbol{n} \tag{13}$$

$$\left[-i\omega\left(1+\alpha\eta^2\right) - \eta^2\right]b = 0 \tag{14}$$

From equation (14), it is evident that the thermal field is uncoupled with purely shear waves. Now, taking the scalar product of equation (13) with $\boldsymbol{a}$, we obtain the following secular equation for purely elastic shear wave in the presence of the rotation of the body:

$$\omega^2\bar{\Omega}^2 - \mu_1\eta^2 = 0 \tag{15}$$

where, $\bar{\Omega}^2 = 1 + q^2\sin^2\phi$, $q^2 = \dfrac{\Omega_0^2}{\omega^2}$

and ϕ is the angle between the directions of $\boldsymbol{\Omega}$ and $\boldsymbol{u}$. We note that in the absence of the rotation of the body, the secular equation (15) reduces to that which holds in the case of a non-rotating body. We also find that equation (15) becomes identical when $\phi = 0$ or π.

The positive root of equation (15) is given by

$$\eta_s = \frac{1}{\sqrt{\mu_1}}\left[\omega^2 + \Omega_0^2\sin^2\phi\right]^{\frac{1}{2}} \tag{16}$$

(I) Phase velocity of shear wave: The phase velocity of wave is defined as

$$V = \frac{\omega}{\mathrm{Re}[\eta]} \tag{17}$$

Therefore, the phase velocity of shear wave is obtained as

$$V_S = \sqrt{\mu_1}\left[1 + \frac{\Omega_0^2 \sin^2\phi}{\omega^2}\right]^{-1/2} \tag{18}$$

Case II: Dilatational waves

For purely dilatational waves $\boldsymbol{u}$ and $\boldsymbol{n}$ have the same directions, so that $\boldsymbol{a}\bullet\boldsymbol{n} = a$, where $a = |\boldsymbol{a}|$ In this case, on taking the scalar product with $\boldsymbol{n}$ of equation (13) equations (13) and (14) become

$$\left(\omega^2 + \Omega_0^2 \sin^2\phi - \eta^2\right)a + ia_1\eta\left(1 + \alpha\eta^2\right)b = 0 \tag{19}$$

$$a_2\eta\omega a + \left[\eta^2 + i\omega\left(1 + \alpha\eta^2\right)\right]b = 0 \tag{20}$$

Equations (19) and (20) clearly imply that the thermal field is coupled with the dilatational wave. Eliminating the constants *a* and *b* from equations (19) and (20), we obtain the following equation for thermoelastic dilatational waves in a rotating body, in the context of the two temperature thermoelasticity theory. For non-trivial solution, the determinant of the coefficient matrix in the above system of equations (19) and (20) must be zero. i.e.

$$\begin{vmatrix} \omega^2 + \Omega_0^2 \sin^2\phi - \eta^2 & ia_1\eta\left(1 + \alpha\eta^2\right) \\ a_2\eta\omega & i\omega\left(1 + \alpha\eta^2\right) + \eta^2 \end{vmatrix} = 0 \tag{21}$$

Therefore, we have a bi-quadratic dispersion relation as

$$\begin{aligned} &\eta^4\left(1 + i\omega\alpha h\right) - \eta^2\left[\Omega_0^2 \sin^2\phi + \omega^2 - i\left\{\omega\left(h - \alpha\Omega_0^2 \sin^2\phi\right) - \omega^3\alpha\right\}\right] \\ &-i\left(\omega^3 + \omega\Omega_0^2 \sin^2\phi\right) = 0 \end{aligned} \tag{22}$$

Now, multiplying above equation throughout by $(1 - i\omega\alpha h)$ we arrive at the simplified form of the dispersion relation as

$$\eta^4\left[1 + \omega^2(\alpha h)^2\right] - \eta^2\left[P - iQ\right] - \left[\omega^4\alpha h + \omega^2\alpha h\Omega_0^2 \sin^2\phi + i\left(\omega\Omega_0^2 \sin^2\phi + \omega^3\right)\right] = 0 \tag{23}$$

where, we have used the following notations:

$P = \Omega_0^2 \sin^2\phi + \omega^2 A_1 + \omega^4\alpha^2 h$, $Q = \omega A_2 + \omega^3\alpha\varepsilon$, $\varepsilon = a_1 a_2$, $h = 1 + \varepsilon$

$A_1 = 1 - \alpha h^2 + \alpha^2 h\Omega_0^2 \sin^2\phi$, $A_2 = h + \alpha\varepsilon\Omega_0^2 \sin^2\phi$.

5. Expressions for attenuation coefficient and wave number of dilatational waves

The roots of equation (23) are $\pm\eta_1$ and $\pm\eta_2$, where $\left(\eta_{1,2}\right)^2 = \dfrac{P - iQ \pm \sqrt{D(\omega)}}{2\left[1+\omega^2\left(\alpha h\right)^2\right]}$ (24)

$$\mathrm{Re}\left[D(\omega)\right] = \Omega_0^4\sin^4\phi + \omega^2\left(2A_1\Omega_0^2\sin^2\phi - A_2^2 + 4\alpha h\Omega_0^2\sin^2\phi\right)$$
$$+\omega^4\left\{A_1^2 + 2\alpha^2 h\Omega_0^2\sin^2\phi - 2A_2\alpha\varepsilon + 4\alpha h + 4\left(\alpha h\right)^3\Omega_0^2\sin^2\phi\right\}$$
$$+\omega^6\left\{2A_1\alpha^2 h - \left(\alpha\varepsilon\right)^2 + 4\left(\alpha h\right)^3\right\} + \omega^8\alpha^4 h^2$$

$$\mathrm{Im}[D(\omega)] = 2\omega\Omega_0^2\sin^2\phi\left(2 - A_2\right) + \omega^3\left[-2A_1A_2 - 2\alpha\varepsilon\Omega_0^2\sin^2\phi + 4 + 4\left(\alpha h\right)^2\Omega_0^2\sin^2\phi\right]$$
$$+\omega^5\left[4\left(\alpha h\right)^2 - 2\left(A_2\alpha^2 h + A_1\alpha\varepsilon\right)\right] - 2\omega^7\alpha^3\varepsilon h$$

It is to be noted here that only two out of the four roots of η given by Eq. (23) have the imaginary parts as negative. We are interested only in these two roots as only these roots yield the negative value of the decay coefficient, $\mathrm{Im}(\eta)$ of the propagating wave. The two values of η with the negative imaginary parts can be obtained from equation (24) by employing the theorem of complex analysis [25, 27]**.** These two values correspond to two different modes of the dilatational wave. One of these is predominately elastic and other is predominately thermal in nature. Let the value of η associated with the former one be denoted by η_1 and the other one by η_2. It should be mentioned here that in absence of rotation (i.e., when $\Omega_0 = 0$) above dispersion relation (23) reduces to the corresponding relation as reported by Puri and Jordan [27].

6. Analytical results

In this section, we will consider two different cases which correspond to the waves of small frequency and waves of high frequency and we will analyze two different modes of dilatational wave in order to find out the effects of rotation on the waves in both the cases.

Special cases

6.1.1. High frequency asymptotic expansions

We consider $\omega >> 1$. Therefore expanding the expressions for $\eta_{1,2}$ from equation (24) in powers of ω^{-1}, we obtain after detailed and long calculations, the asymptotic expressions of η_1 and η_2 for high frequency values as follows:

$$\eta_1 \approx \frac{\omega}{\sqrt{h}}\left[1+\frac{1}{\omega^2}\left(\frac{\varepsilon(\varepsilon+4)}{8(\alpha h)^2}+\frac{\Omega_0^2\sin^2\phi}{2}\right)-\frac{i\varepsilon}{2\omega\alpha h}\left\{1-\frac{1}{\omega^2}\left(\frac{4\alpha h^3-\varepsilon^2+2\varepsilon+4-2\,\Omega_0^2\sin^2\phi}{4(\alpha h)^2}\right)\right\}\right] \quad (25)$$

$$\eta_2 \approx \frac{1}{\sqrt{\alpha}}\left[\frac{1}{2\omega\alpha}\left\{1+\frac{1}{2\omega^2}\left(\frac{1+4\varepsilon}{\alpha h^4}-\frac{3\varepsilon^2\Omega_0^2\sin^2\phi}{4h^3}\right)^2\right\}-i\left\{1-\frac{1}{\omega^2}\left(\frac{3}{8\alpha^2}-\frac{\varepsilon\,\Omega_0^2\sin^2\phi}{8\alpha}\right)\right\}\right] \quad (26)$$

6.1.2. Low–frequency asymptotic expansions

We consider $\omega << 1$. Expanding the expressions for $\eta_{1,2}$ from equation (24) in powers of ω we obtain after detailed and long calculations the asymptotic expansions of η_1 and η_2 for low frequency values as follows:

$$\eta_1 \approx \Omega_0\sin\phi\left[1+\omega^2\left(\frac{-\alpha^2\varepsilon(4+3\varepsilon)}{8}+\frac{2\Omega_0^2\sin^2\phi\left\{2+\alpha\left(2+\varepsilon^2-2\alpha h\right)\right\}+\varepsilon(\varepsilon-4)}{8\Omega_0^4\sin^4\phi}\right)-i\frac{\omega\varepsilon}{2}\left(\frac{1}{\Omega_0^2\sin^2\phi}+\alpha\right)\right] \quad (27)$$

$$\eta_2 \approx \begin{cases}\sqrt{\dfrac{\omega}{2}}\left[1+\omega\left\{\dfrac{\left(\varepsilon-\alpha\ \Omega_0^2\sin^2\phi\right)}{2\Omega_0^2\sin^2\phi}\right\}-i\left\{1-\omega\dfrac{\left(\varepsilon-\alpha\ \Omega_0^2\sin^2\phi\right)}{2\Omega_0^2\sin^2\phi}\right\}\right],\ \alpha\neq\alpha^* \\ \sqrt{\dfrac{\omega}{2}}\left[1-\omega^2\left\{\dfrac{\varepsilon(\varepsilon-2)-\alpha\,\Omega_0^2\sin^2\phi\left(4\varepsilon\ -\alpha\,\Omega_0^2\sin^2\phi\right)}{4\Omega_0^4\sin^4\phi}\right\}-i\left\{1-\omega^2\left\{\dfrac{\varepsilon(\varepsilon-2)-\alpha\,\Omega_0^2\sin^2\phi\left(4\varepsilon\ -\alpha\,\Omega_0^2\sin^2\phi\right)}{4\Omega_0^4\sin^4\phi}\right\}\right\}\right],\alpha=\alpha^*\end{cases}$$

(28)

where $\alpha^* = \dfrac{\varepsilon}{\Omega_0^2\sin^2\phi}$ is the critical value of the two-temperature parameter. Clearly the critical value depends on the thermoelastic coupling constant as well as on the magnitude of rotation.

a. **Asymptotic results of different wave fields**

We obtain the asymptotic expressions of different characterizations of wave fields such as phase velocity, specific loss, penetration depth etc. of both the modified elastic and modified thermal mode dilatational waves and examine these quantities under the cases of high frequency values as well as low frequency values.

(I) Phase velocity: The phase velocity is given by

$$V_{E,T} = V_{1,2} = \frac{\omega}{\mathrm{Re}\left[\eta_{1,2}\right]} \tag{29}$$

where V_E is the velocity of elastic mode dilatational wave and V_T is the velocity of the thermal mode dilatational wave. Now we obtain the asymptotic expansions of these phase velocities from Eqs. (25) – (28) and the formula (29) as follows:

High frequency asymptotics

$$V_E \approx \sqrt{h}\left[1 - \frac{1}{\omega^2}\left(\frac{\varepsilon(\varepsilon+4)}{8(\alpha h)^2} + \frac{\Omega_0^2 \sin^2\phi}{2}\right)\right], \qquad (\omega \to \infty) \tag{30}$$

$$V_T \approx 2\omega^2 (\alpha)^{3/2}\left[1 - \frac{1}{2\omega^2}\left(\frac{1+4\varepsilon}{\alpha h^4} - \frac{3\varepsilon^2\Omega_0^2 \sin^2\phi}{4h^3}\right)^2\right], \ (\omega \to \infty) \tag{31}$$

Low frequency asymptotics

$$V_E \approx \frac{\omega}{\Omega_0 \sin\phi}\left[1 - \omega^2\left(\frac{-\alpha^2\varepsilon(4+3\varepsilon)}{8} + \frac{2\Omega_0^2\sin^2\phi\left\{2+\alpha\left(2+\varepsilon^2-2\alpha h\right)\right\}+\varepsilon(\varepsilon-4)}{8\Omega_0^4\sin^4\phi}\right)\right], \ (\omega \to 0) \tag{32}$$

$$V_T \approx \begin{cases} \sqrt{2\omega}\left[1-\omega\left\{\dfrac{\left(\varepsilon-\alpha\Omega_0^2\sin^2\phi\right)^2}{2\Omega_0^2\sin^2\phi}\right\}\right], \ \alpha \neq \alpha^* \\[2ex] \sqrt{2\omega}\left[1+\omega^2\left\{\dfrac{\varepsilon(\varepsilon-2)-\alpha\Omega_0^2\sin^2\phi\left(4\varepsilon-\alpha\Omega_0^2\sin^2\phi\right)}{4\Omega_0^4\sin^4\phi}\right\}\right], \ \alpha = \alpha^* \end{cases} \qquad (\omega \to 0) \tag{33}$$

(II) Specific loss: The specific loss of wave is defend as the ratio of energy dissipated per stress cycle to the total vibrational energy and is given by

$$\left(\frac{\Delta W}{W}\right)_{E,T} = \left(\frac{\Delta W}{W}\right)_{1,2} = 4\pi\left|\frac{\mathrm{Im}\left[\eta_{1,2}\right]}{\mathrm{Re}\left[\eta_{1,2}\right]}\right| \tag{34}$$

Eqs.(25) - (28) and the formula given in (34) yield the high frequency and low frequency asymptotic expressions for the specific loss of both the modes of dilatational wave as

High frequency asymptotics

$$\left(\frac{\Delta W}{W}\right)_E \approx \frac{2\pi\varepsilon}{\omega\alpha h}\left|1-\frac{1}{\omega^2}\left(\frac{4\alpha h^3-\varepsilon^2+2\varepsilon+4}{4(\alpha h)^2}-\frac{1}{2}\,\Omega_0^2\sin^2\phi\right)\right|, \qquad (\omega\to\infty) \tag{35}$$

$$\left(\frac{\Delta W}{W}\right)_T \approx 8\pi\omega\alpha\left|1+\frac{1}{\omega^2}\left(\frac{3}{8\alpha^2}-\frac{\varepsilon\,\Omega_0^2\sin^2\phi}{8\alpha}\right)\right|, \qquad (\omega\to\infty) \tag{36}$$

Low frequency asymptotics

$$\left(\frac{\Delta W}{W}\right)_E \approx 2\pi\,\omega\,\varepsilon\left(\alpha+\frac{1}{\Omega_0^2\sin^2\phi}\right)\left[1-\omega^2\left(\frac{-\alpha^2\varepsilon(4+3\varepsilon)}{8}+\frac{2\Omega_0^2\sin^2\phi\left\{2+\alpha\left(2+\varepsilon^2-2\alpha h\right)\right\}+\varepsilon(\varepsilon-4)}{8\Omega_0^4\sin^4\phi}\right)\right], \quad (\omega\to 0) \tag{37}$$

$$\left(\frac{\Delta W}{W}\right)_T \approx \begin{cases} 4\pi\left[1-\omega\left(\dfrac{\varepsilon-\alpha\,\Omega_0^2\sin^2\phi}{\Omega_0^2\sin^2\phi}\right)\right], & \alpha\neq\alpha^* \\[2ex] 4\pi\left[1-\omega^2\left\{\dfrac{\varepsilon(\varepsilon-2)-\alpha\,\Omega_0^2\sin^2\phi\left(4\varepsilon-\alpha\,\Omega_0^2\sin^2\phi\right)}{2\,\Omega_0^4\sin^4\phi}\right\}\right], & \alpha=\alpha^* \end{cases} \qquad (\omega\to 0) \tag{38}$$

(III) Penetration depth: The penetration depth is defined as

$$\delta_{E,T}=\left|\frac{1}{\mathrm{Im}\left[\eta_{1,2}\right]}\right| \tag{39}$$

Therefore from Eqs. (25) - (28) and formula (39), we find the high frequency and low frequency asymptotic expansions for the penetration depth as follows:

High frequency asymptotics

$$\delta_E \approx \frac{2\alpha(h)^{3/2}}{\varepsilon}\left[1+\frac{1}{\omega^2}\left(\frac{4\alpha h^3-\varepsilon^2+2\varepsilon+4}{4(\alpha h)^2}-\frac{1}{2}\,\Omega_0^2\sin^2\phi\right)\right], \qquad (\omega\to\infty) \tag{40}$$

$$\delta_T \approx \sqrt{\alpha}\left[1-\frac{1}{\omega^2}\left(\frac{3}{8\alpha^2}-\frac{\varepsilon\,\Omega_0^2\sin^2\phi}{8\alpha}\right)\right], \qquad (\omega\to\infty) \tag{41}$$

Low frequency asymptotics

$$\delta_E \approx \frac{2\Omega_0 \sin\phi}{\omega\varepsilon\left(1+\alpha\,\Omega_0^2 \sin^2\phi\right)}\ , \qquad (\omega \to 0) \tag{42}$$

$$\delta_T \approx \begin{cases} \sqrt{\dfrac{2}{\omega}}\left[1+\omega\left(\dfrac{\varepsilon-\alpha\,\Omega_0^2\sin^2\phi}{2\Omega_0^2\sin^2\phi}\right)\right],\ \alpha \neq \alpha^* \\ \sqrt{\dfrac{2}{\omega}}\left[1+\omega^2\left\{\dfrac{\varepsilon(\varepsilon-2)-\alpha\,\Omega_0^2\sin^2\phi\left(4\varepsilon\ -\alpha\,\Omega_0^2\sin^2\phi\right)}{4\Omega_0^4\sin^4\phi}\right\}\right],\ \alpha = \alpha^* \end{cases}, \quad (\omega \to 0) \tag{43}$$

(V) Amplitude coefficient factor and phase shift of thermodynamic temperature

Using Equations (10) and (5) we can write the thermodynamic temperature as

$$\theta(x,t) = (1+\alpha\eta^2)b\exp\left[i\left(\omega t-\eta\boldsymbol{n}\bullet x\right)\right] = (1+\alpha\eta^2)\varphi = Mexp(i\psi)\varphi \tag{44}$$

where, $M = \left|1+\alpha\eta^2\right|\ ,\ \ \psi = Arg\left(1+\alpha\eta^2\right)$ (45)

Here M is termed as the amplitude coefficient factor and ψ is the phase-shift. Now according to the expressions given by Eq. (45) we obtain the asymptotic expansions of these quantities as follows:

High frequency asymptotes

$$M_E \approx \frac{\alpha\omega^2}{h}\left[1+\frac{1}{\omega^2}\left(\frac{h}{\alpha}+\varepsilon\frac{(\varepsilon+4)}{4(\alpha h)^2}+\Omega_0^2\sin^2\phi\right)\right], \qquad (\omega\to\infty) \tag{46}$$

$$\psi_E \approx -\frac{\varepsilon}{\alpha h\omega}\ , \qquad (\omega\to\infty) \tag{47}$$

$$M_T \approx \frac{1}{\omega\alpha}\left[1+\frac{1}{2\omega^2}\left(\frac{9\varepsilon\,\Omega_0^2\sin^2\phi}{4}\left(\frac{9}{\alpha}+\frac{\varepsilon\,\Omega_0^2\sin^2\phi}{4}\right)-\left(\frac{9-64\varepsilon}{16\alpha^2}\right)\right)\right], \qquad (\omega\to\infty) \tag{48}$$

$$\psi_T \approx\ -\frac{\pi}{2}\ , \quad (\omega\to\infty) \tag{49}$$

Low frequency asymptotes

$$M_{\mathrm{E}} \approx \left(1+\alpha\,\Omega_0^2\sin^2\phi\right)\left[1+\omega^2\left\{\frac{\alpha\,\Omega_0^2\sin^2\phi\left(2c_1-c_2^2\right)}{\left(1+\alpha\,\Omega_0^2\sin^2\phi\right)}+\left(\frac{\alpha\varepsilon}{2}\right)^2\right\}\right]\ ,\ (\omega\to 0) \tag{50}$$

where $c_1=\left(\dfrac{-\alpha^2\varepsilon\left(4+3\varepsilon\right)}{8}+\dfrac{2\Omega_0^2\sin^2\phi\left\{2+\alpha\left(2+\varepsilon^2-2\alpha h\right)\right\}+\varepsilon\left(\varepsilon-4\right)}{8\Omega_0^4\sin^4\phi}\right)$

$$c_2=\frac{\varepsilon\left(1+\alpha\,\Omega_0^2\sin^2\phi\right)}{2\Omega_0^2\sin^2\phi}$$

$$\psi_E \approx -\omega\alpha\varepsilon \quad , \qquad (\omega\to 0) \tag{51}$$

$$M_T \approx \begin{cases} 1+\left(\dfrac{\alpha\varepsilon}{\Omega_0^2\sin^2\phi}-\dfrac{\alpha^2}{2}\right)\omega^2, \ \alpha\neq\alpha^* \\ 1+\dfrac{(\alpha\omega)^2}{2}-\alpha^2\omega^4\left\{\dfrac{\varepsilon\left(\varepsilon-2\right)-\alpha\,\Omega_0^2\sin^2\phi\left(4\varepsilon-\alpha\,\Omega_0^2\sin^2\phi\right)}{2\Omega_0^4\sin^4\phi}\right\}, \ \alpha=\alpha^* \end{cases} \quad , \quad (\omega\to 0) \tag{52}$$

$$\psi_T \approx \begin{cases} -\alpha\omega, \qquad \alpha\neq\alpha^* \\ -\alpha\omega\left[1-\left\{\dfrac{\varepsilon\left(\varepsilon-2\right)-\alpha\,\Omega_0^2\sin^2\phi\left(4\varepsilon-\alpha\,\Omega_0^2\sin^2\phi\right)}{2\Omega_0^4\sin^4\phi}\right\}\omega^2\right], \ \alpha=\alpha^* \end{cases} \ , \quad (\omega\to 0) \tag{53}$$

7. Numerical results

In this section, we illustrate the asymptotic results obtained in the previous sections in order to examine the behavior of phase velocity, specific loss, penetration depth, amplitude coefficient factor and phase shift of the thermodynamic temperature due to rotation. For computational work, we assume $\alpha=0.071301, \Omega_0=0.01$ and $\varepsilon=0.0168$. Using the formulae given by (29), (34), (39) and (45) we compute the values of the quantities directly from Eq. (23) for various values of frequency, ω and the rotational angle, ϕ. We also compute the numerical values of different characterizations for the case of absence of rotation by putting $\phi=0$. We have plotted the results in different Figures.

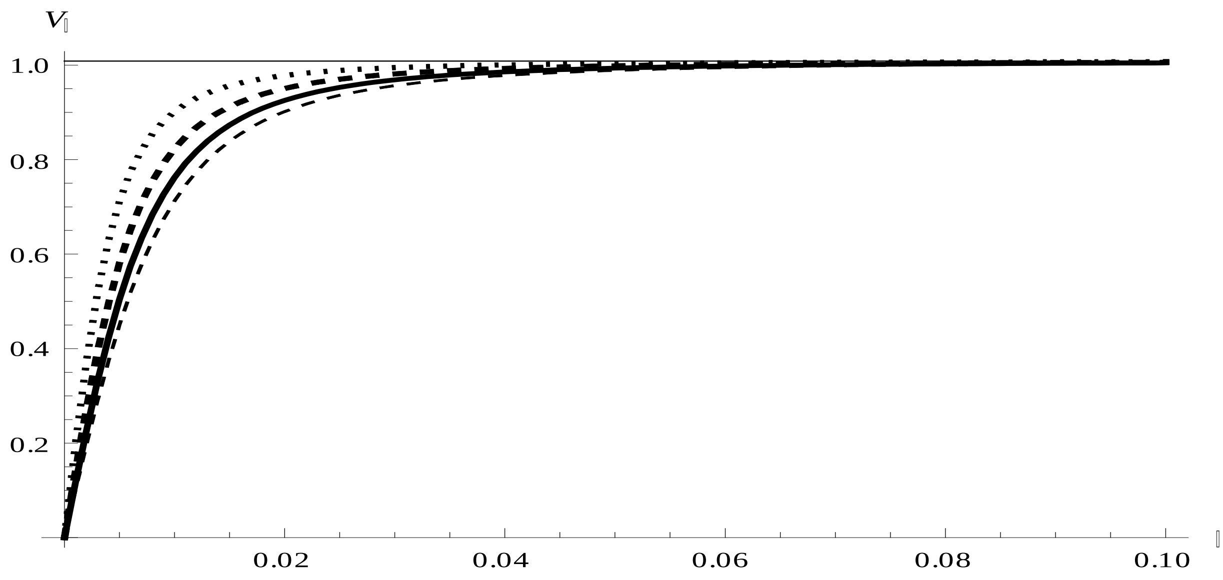

Fig. 1(a): V_E **Vs** ω **-** $\phi = 0$ **: Thin solid line,** $\phi = \dfrac{\pi}{6}$ **: Thick dotted line,** $\phi = \dfrac{\pi}{4}$ **: Thick dashed line,** $\phi = \dfrac{\pi}{3}$ **: Thick solid line,** $\phi = \dfrac{\pi}{2}$ **: Thin dashed line.**

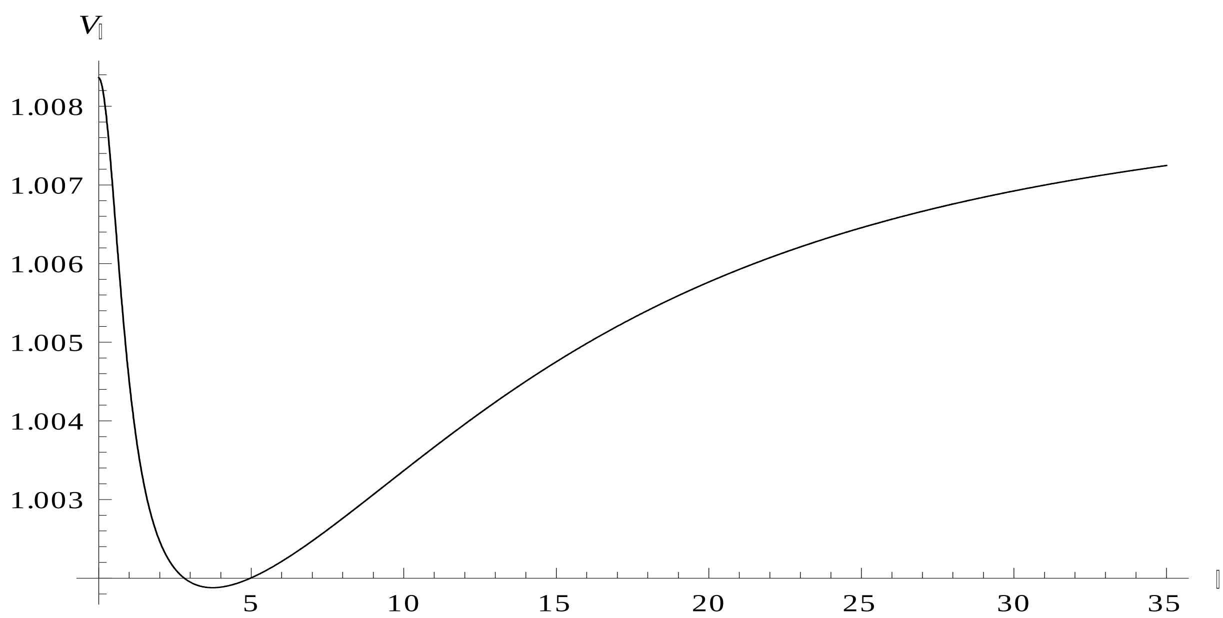

Fig. 1 (b): V_E **Vs** ω **for** $\phi = 0$

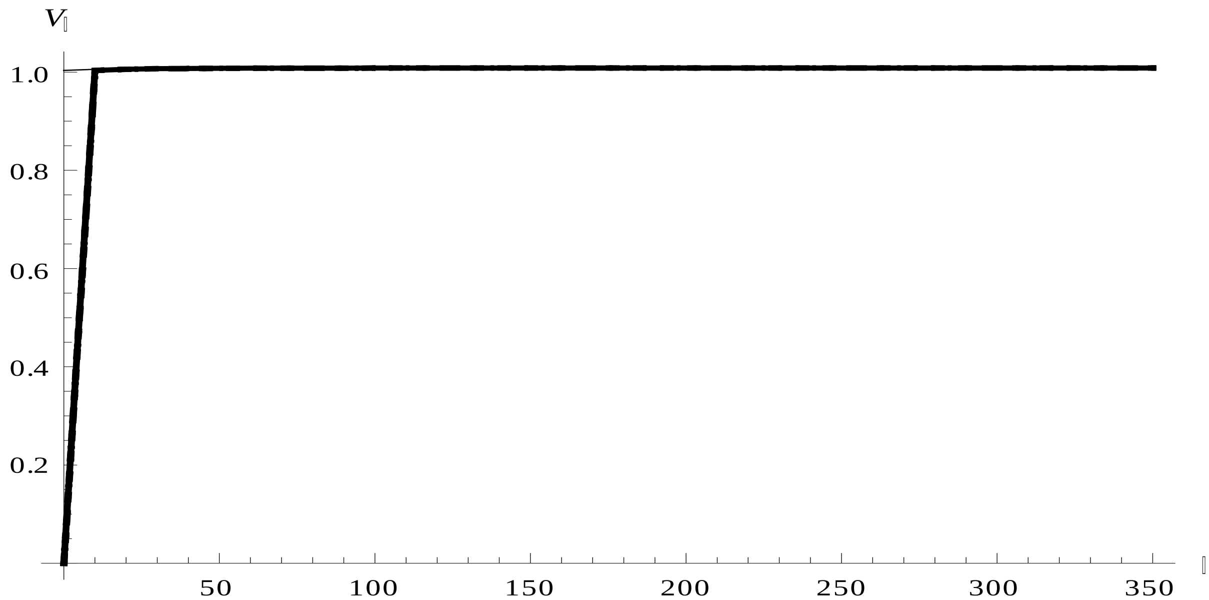

Fig. 1(c): V_E **Vs** ω **-** $\phi = 0$ **: Thin solid line,** $\phi = \frac{\pi}{6}$ **: Thick dotted line,** $\phi = \frac{\pi}{4}$ **: Thick dashed line,** $\phi = \frac{\pi}{3}$ **: Thick solid line,** $\phi = \frac{\pi}{2}$ **: Thin dashed line.** (All the curves for the case of $\phi \neq 0$ are merged together.)

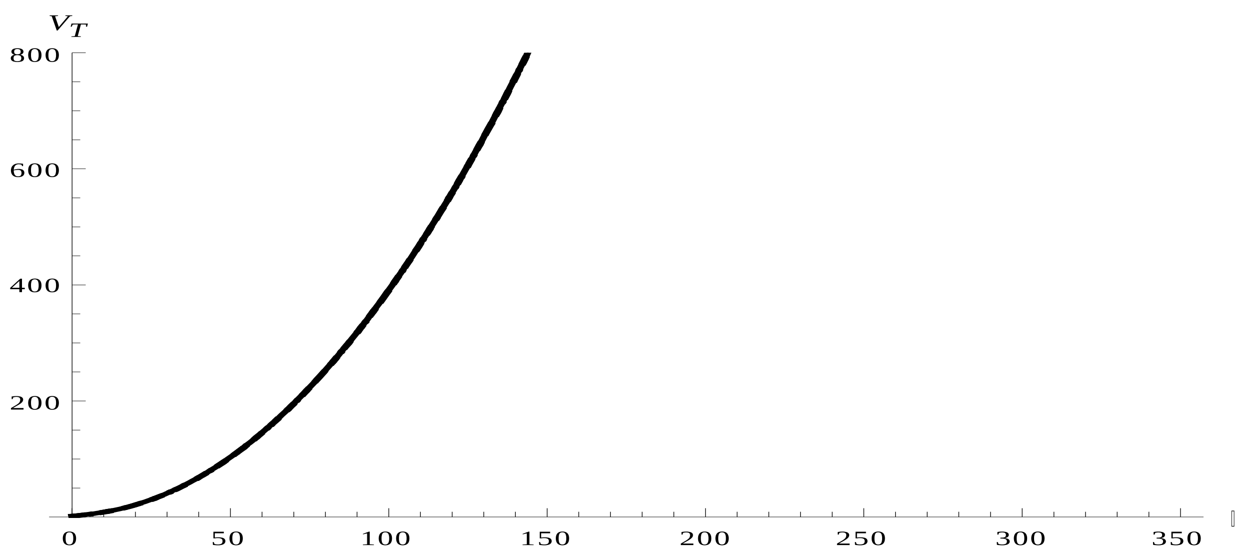

Fig. 1(d) : V_T **Vs** ω **-** $\phi = 0$ **: Thin solid line,** $\phi = \frac{\pi}{6}$ **: Thick dotted line,** $\phi = \frac{\pi}{4}$ **: Thick dashed line,** $\phi = \frac{\pi}{3}$ **: Thick solid line,** $\phi = \frac{\pi}{2}$ **: Thin dashed line.** (All the curves are merged together.)

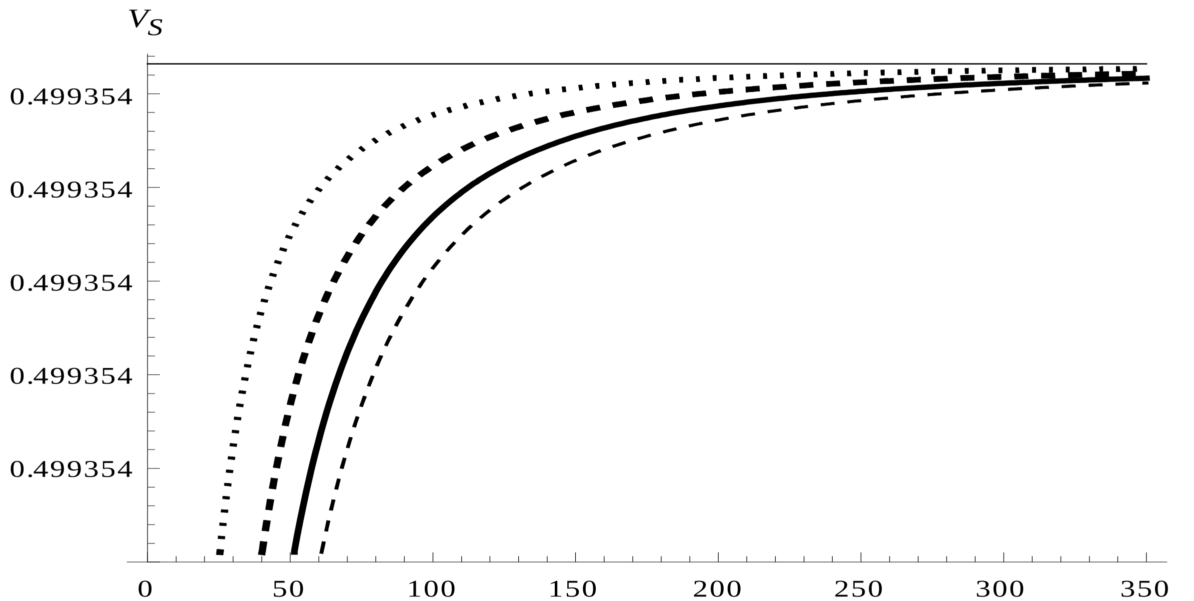

Fig. 1(e): V_S **Vs** ω **-** $\phi = 0$**: Thin solid line,** $\phi = \frac{\pi}{6}$**: Thick dotted line,** $\phi = \frac{\pi}{4}$**: Thick dashed line,** $\phi = \frac{\pi}{3}$**: Thick solid line,** $\phi = \frac{\pi}{2}$**: Thin dashed line.**

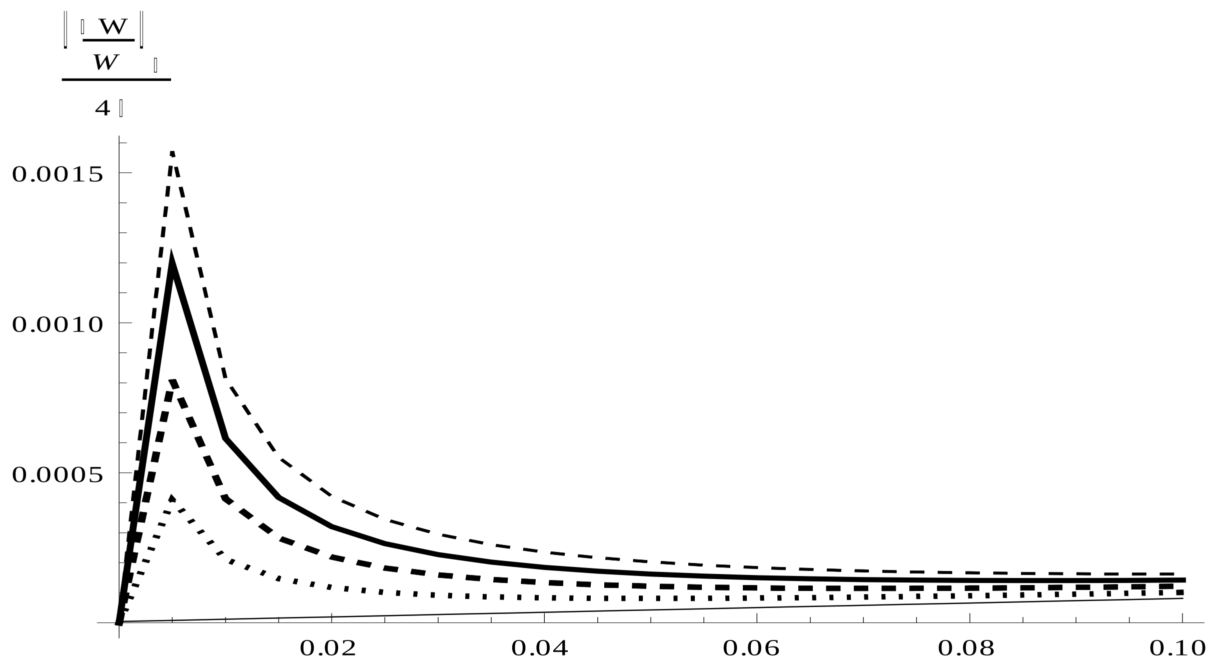

Fig. 2(a): $\left(\frac{\Delta W}{W}\right)_E$ **Vs** ω **-** $\phi = 0$**: Thin solid line,** $\phi = \frac{\pi}{6}$**: Thick dotted line,** $\phi = \frac{\pi}{4}$**: Thick dashed line,** $\phi = \frac{\pi}{3}$**: Thick solid line,** $\phi = \frac{\pi}{2}$**: Thin dashed line.**

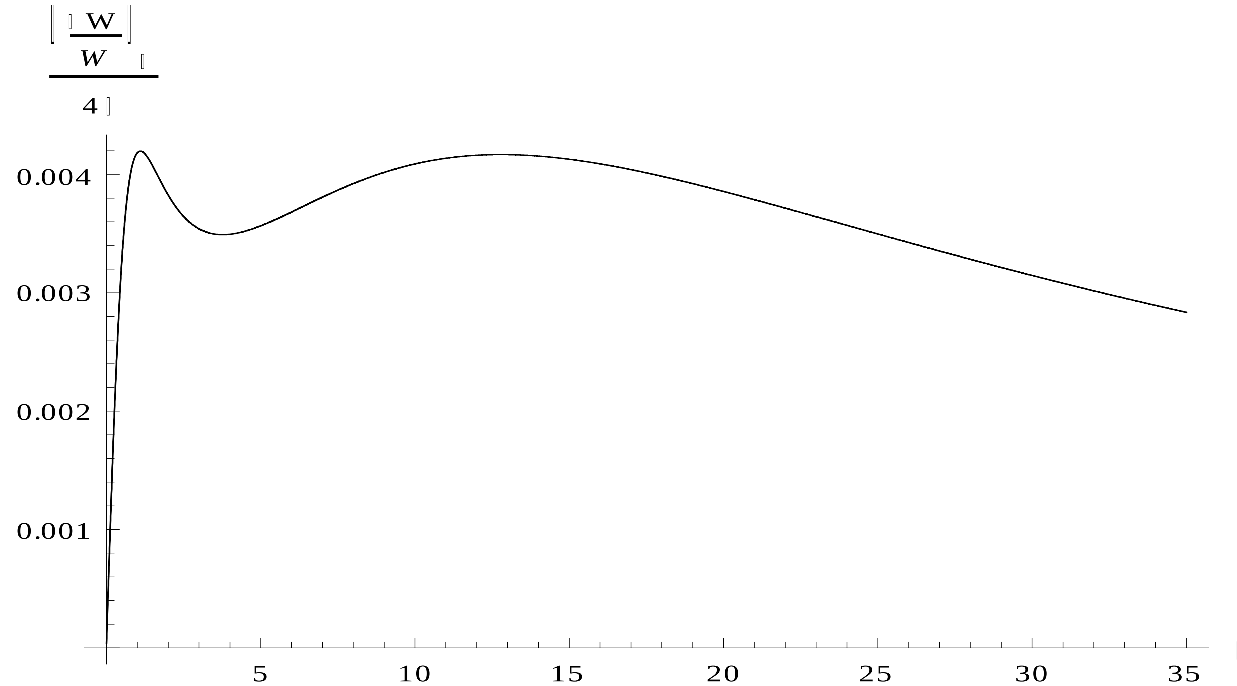

Fig. 2(b) : $\left(\frac{\Delta W}{W}\right)_E$ **Vs** ω **for** $\phi = 0$

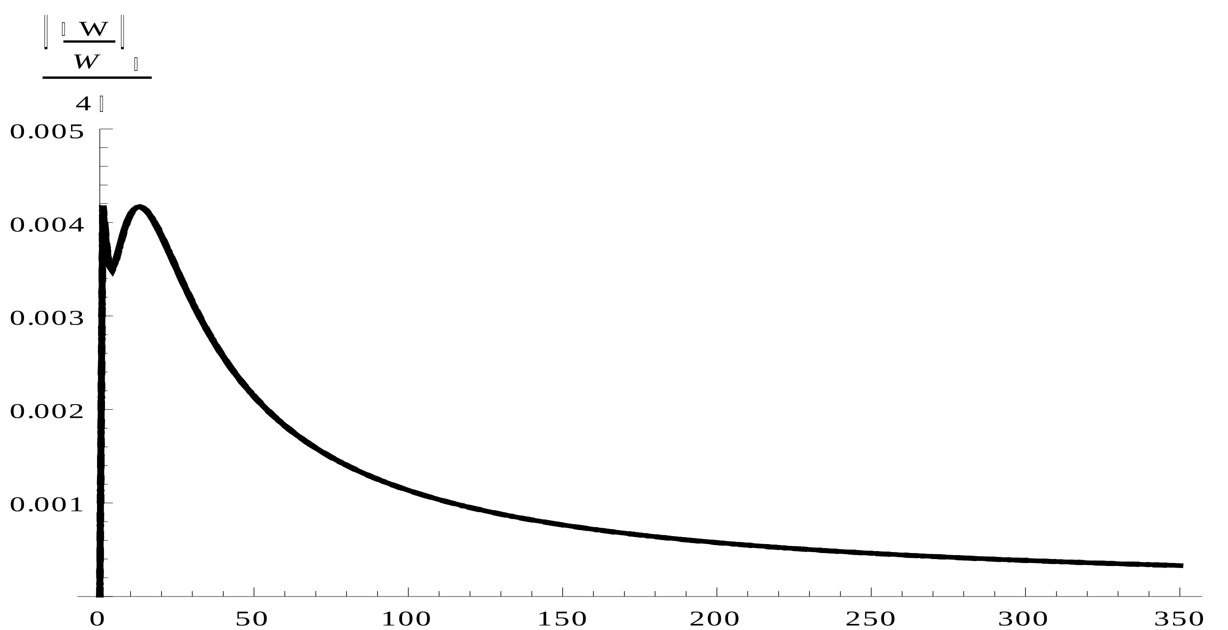

Fig. 2(c): $\left(\frac{\Delta W}{W}\right)_E$ **Vs** ω - $\phi = 0$**: Thin solid line,** $\phi = \frac{\pi}{6}$**: Thick dotted line,** $\phi = \frac{\pi}{4}$ **: Thick dashed line,** $\phi = \frac{\pi}{3}$ **: Thick solid line,** $\phi = \frac{\pi}{2}$**: Thin dashed line.** (All the curves are merged together.)

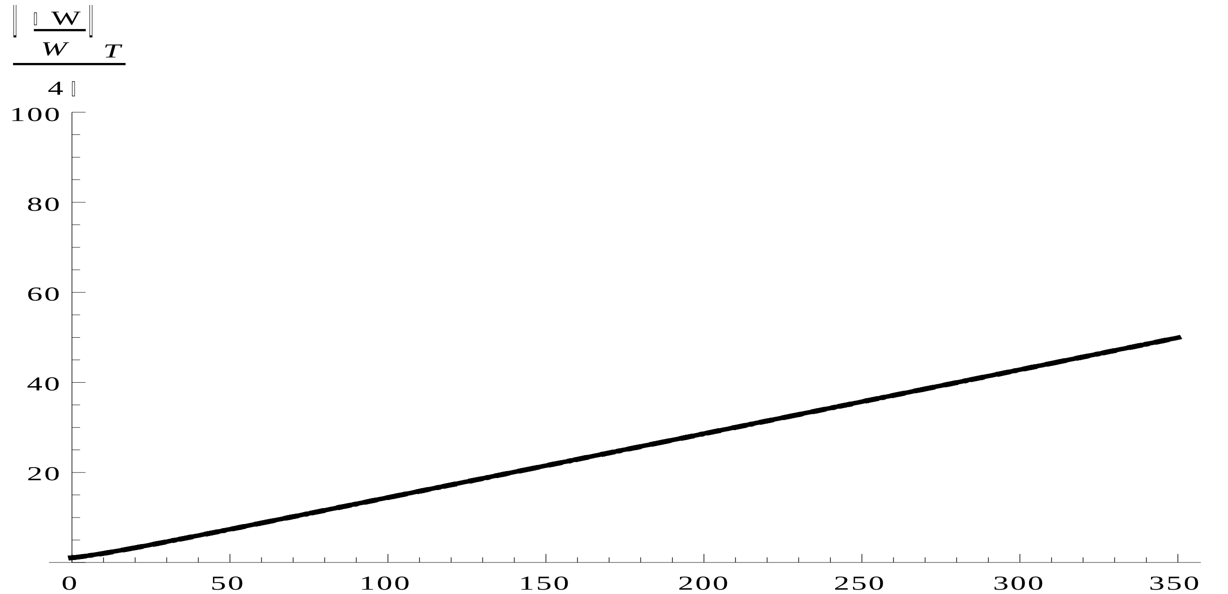

Fig. 2(d): $\left(\frac{\Delta W}{W}\right)_T$ **Vs** ω **-** $\phi = 0$ **: Thin solid line,** $\phi = \frac{\pi}{6}$ **: Thick dotted line,** $\phi = \frac{\pi}{4}$ **: Thick dashed line,** $\phi = \frac{\pi}{3}$ **: Thick solid line,** $\phi = \frac{\pi}{2}$ **: Thin dashed line.** (All the curves are merged together.)

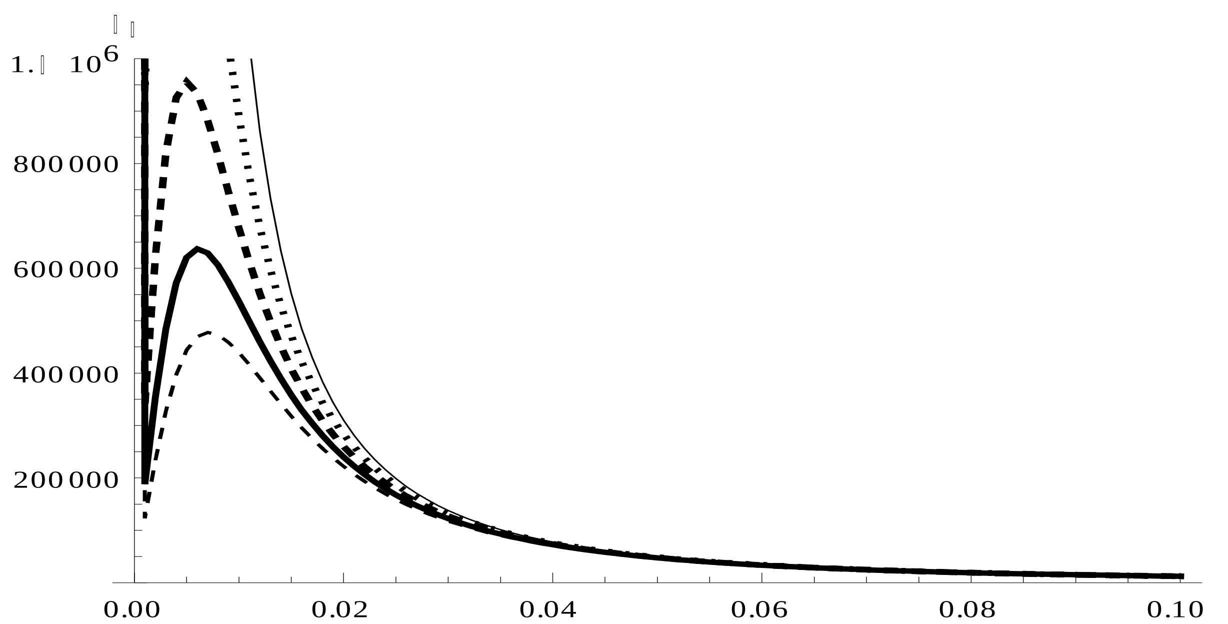

Fig. 3(a): δ_E **Vs** ω **-** $\phi = 0$ **: Thin solid line,** $\phi = \frac{\pi}{6}$ **: Thick dotted line,** $\phi = \frac{\pi}{4}$ **: Thick dashed line,** $\phi = \frac{\pi}{3}$ **: Thick solid line,** $\phi = \frac{\pi}{2}$ **: Thin dashed line.**

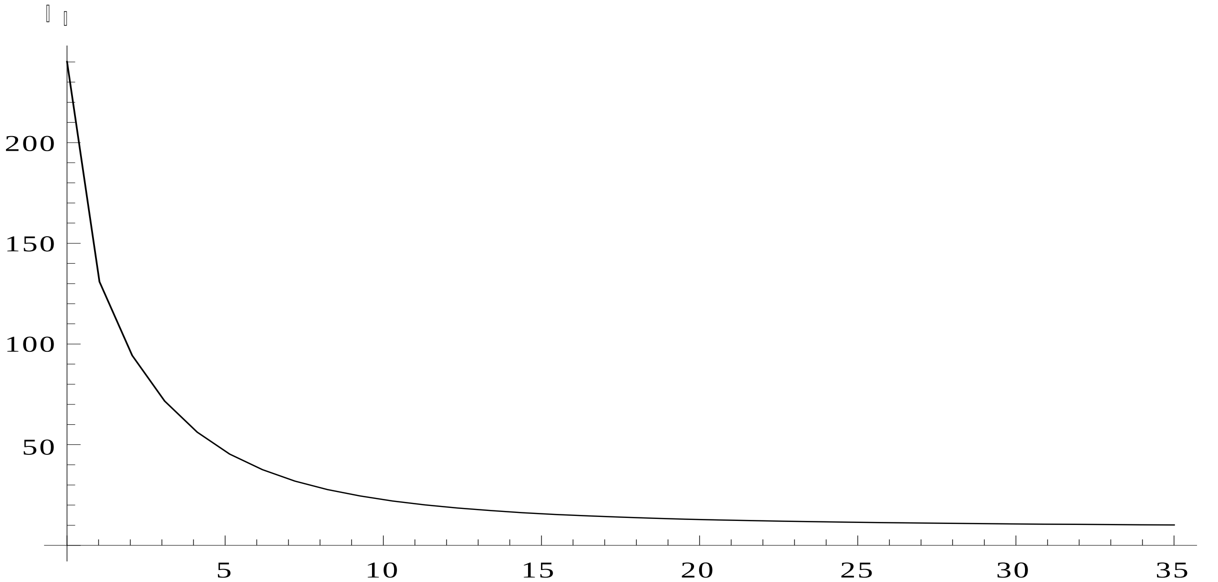

Fig. 3(b): δ_E **Vs** ω $\phi = 0$

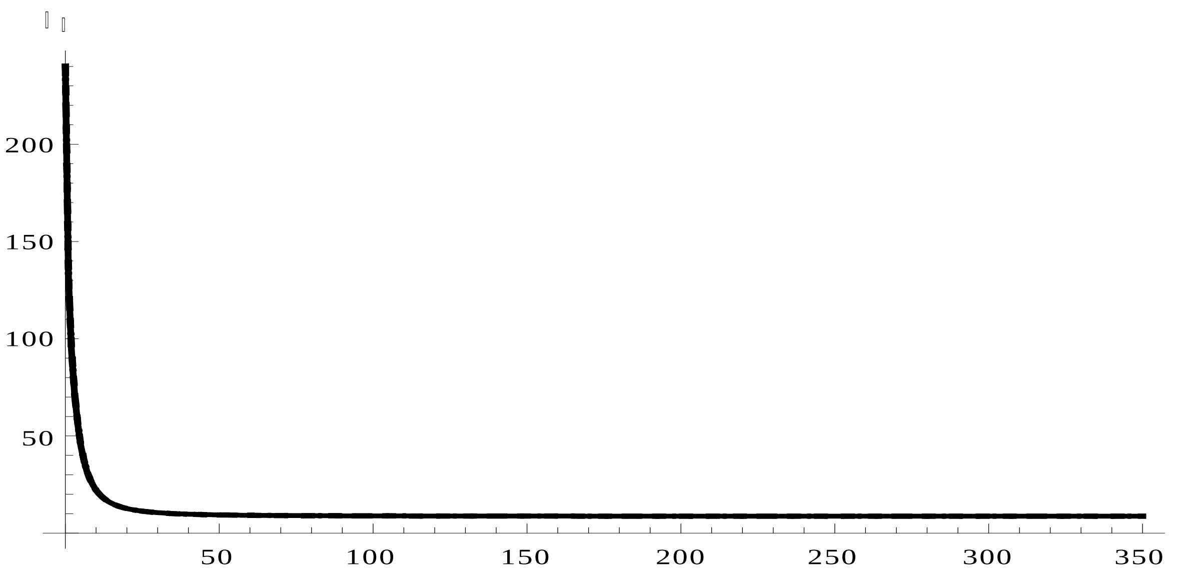

Fig. 3(c): δ_E **Vs** ω - $\phi = 0$ **: Thin solid line,** $\phi = \dfrac{\pi}{6}$ **: Thick dotted line,** $\phi = \dfrac{\pi}{4}$ **: Thick dashed line,** $\phi = \dfrac{\pi}{3}$ **: Thick solid line,** $\phi = \dfrac{\pi}{2}$ **: Thin dashed line.** (All the curves are merged together.)

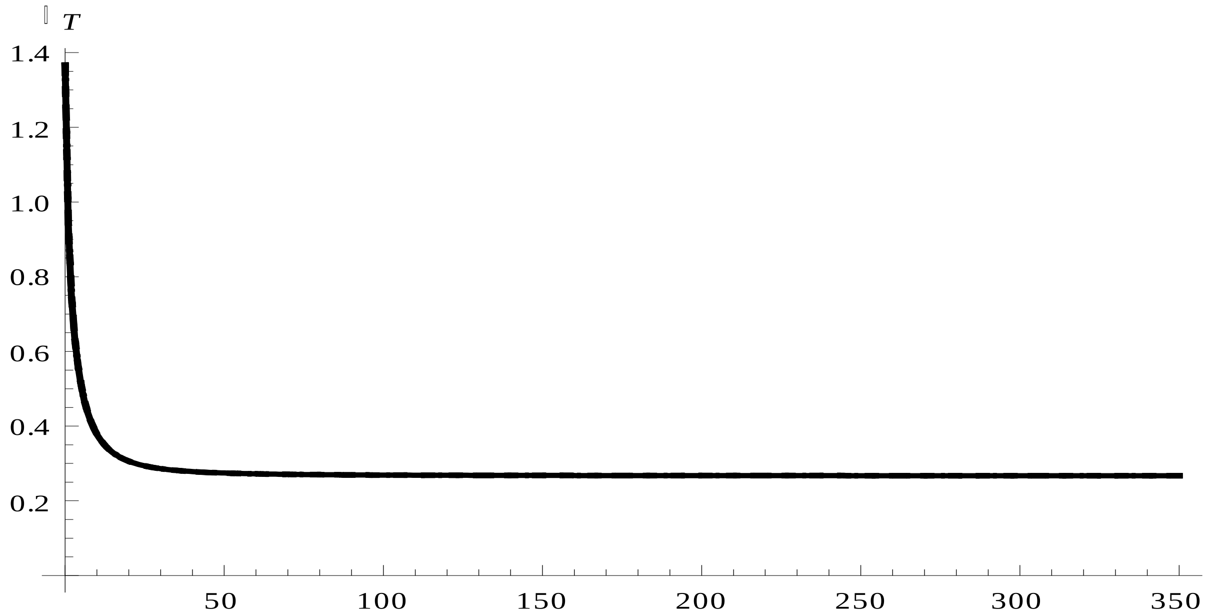

Fig. 3(d): δ_T **Vs** ω - $\phi = 0$ **: Thin solid line,** $\phi = \dfrac{\pi}{6}$ **: Thick dotted line,** $\phi = \dfrac{\pi}{4}$ **: Thick dashed line,** $\phi = \dfrac{\pi}{3}$ **: Thick solid line,** $\phi = \dfrac{\pi}{2}$ **: Thin dashed line.** (All the curves are merged together.)

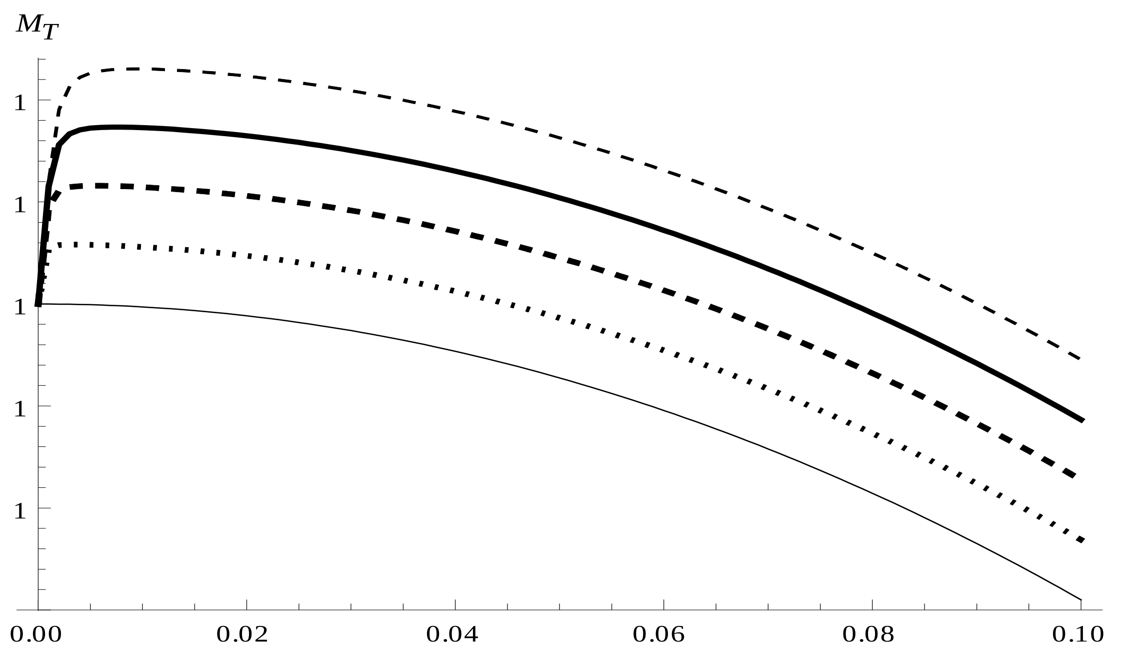

Fig. 4(a): M_T **Vs** ω - $\phi = 0$ **: Thin solid line,** $\phi = \dfrac{\pi}{6}$ **: Thick dotted line,** $\phi = \dfrac{\pi}{4}$ **: Thick dashed line,** $\phi = \dfrac{\pi}{3}$ **: Thick solid line,** $\phi = \dfrac{\pi}{2}$ **: Thin dashed line.** (All the curves are merged together.)

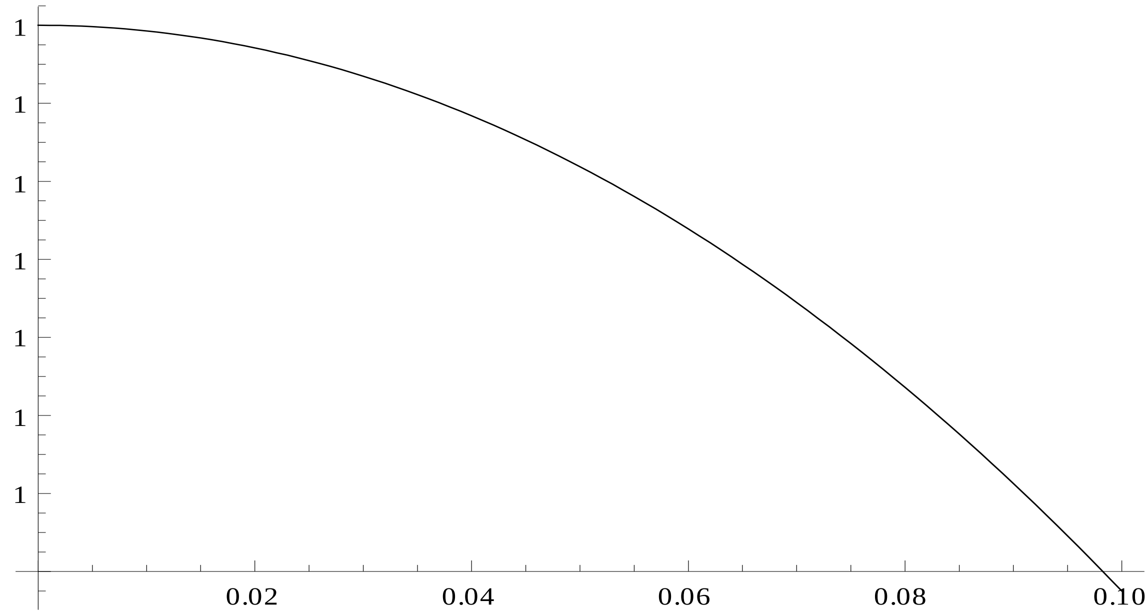

Fig. 4(b): M_T **Vs** $\omega\ \phi = 0$

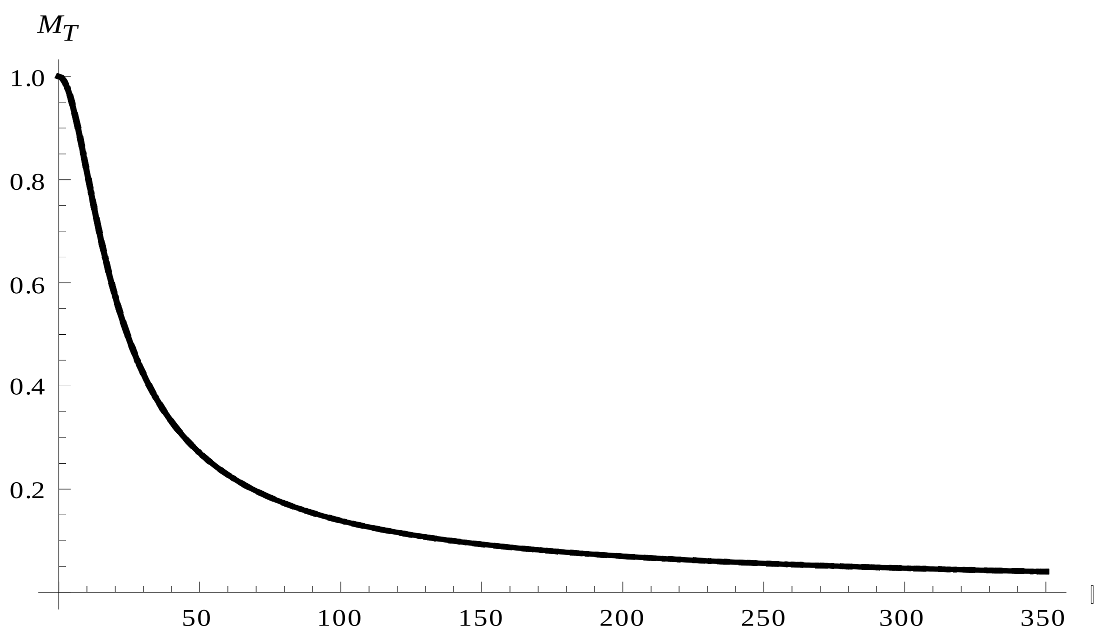

Fig. 4(c): M_T **Vs** ω - $\phi = 0$ **: Thin solid line,** $\phi = \dfrac{\pi}{6}$ **: Thick dotted line,** $\phi = \dfrac{\pi}{4}$ **: Thick dashed line,** $\phi = \dfrac{\pi}{3}$ **: Thick solid line,** $\phi = \dfrac{\pi}{2}$ **: Thin dashed line.** (All the curves are merged together.)

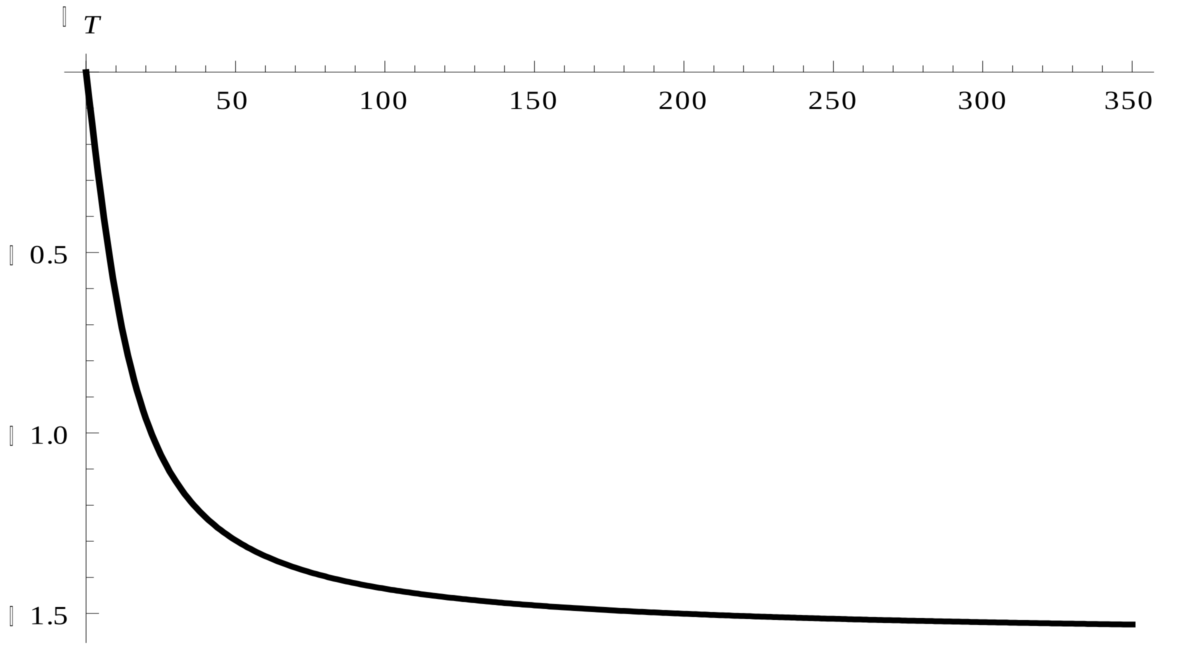

Fig. 4(d): ψ_T **Vs** ω - $\phi = 0$ **: Thin solid line,** $\phi = \dfrac{\pi}{6}$ **: Thick dotted line,** $\phi = \dfrac{\pi}{4}$ **: Thick dashed line,** $\phi = \dfrac{\pi}{3}$ **: Thick solid line,** $\phi = \dfrac{\pi}{2}$ **: Thin dashed line.** (All the curves are merged together.)

Summary and observations

Harmonic plane waves propagating in a rotating elastic medium under two-temperature thermoelasticity theory are investigated. Dispersion relation solutions of longitudinal as well as transverse plane waves are determined. The transverse wave is observed to be unaffected due to thermal field. High and low frequency asymptotics of different wave characterizations for longitudinal elastic (predominated) and thermal waves (predominated) are found out and detail analysis of the results highlighting the effects of rotation is presented with the help of different graphs. The following important facts are observed:

(1) There are elastic shear wave and predominantly elastic and thermal mode dilatational waves for all $\omega > 0$. All waves are dispersive in nature.

(2) There exists a critical value of two-temperature parameter given by $\alpha^* = \dfrac{\varepsilon}{\Omega_0^2 \sin^2\phi}$ and this critical value is clearly effected due to rotation.

(3) Effects of rotation on shear wave and elastic dilatational wave is prominent for lower values of ω. But effects on thermal wave mode longitudinal wave is negligible.

(4) Phase velocity decreases and the specific loss increases with the increase of rotational angle, ϕ. The phase velocity of elastic mode wave shows a local minimum in absence of rotation but in presence rotation there is no such extreme value.

(5) The penetration depth for elastic wave has a constant limiting value $\frac{2\alpha h^{\frac{3}{2}}}{\varepsilon}$ as $\omega \to \infty$ in presence of rotation and similar result is found in absence of rotation (see ref. (27)). But in presence of rotation this profile shows two extreme values (one minimum and one maximum). δ_T tends to the constant limiting value $\sqrt{\alpha}$ as $\omega \to \infty$ in all cases.

(6) We note that in all cases $|M_T b| < |b|$, i.e. the thermodynamic temperature θ exhibits a lesser magnitude as compared to the conductive temperature and it experiences a phase shift $\psi_T < 0$, where $\lim_{\omega \to 0} \psi_T = 0$ and $\lim_{\omega \to \infty} \psi_T = -\frac{\pi}{2}$.

(7) The limiting values of all quantities when $\omega \to \infty$ are exactly the same in all cases and therefore no effect of rotation is observed for higher values of frequency. As $\omega \to 0$, the limiting values of all quantities except V_E are also found to be the same.

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