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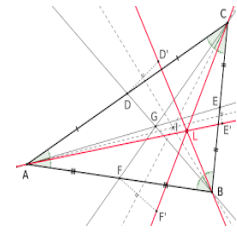


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IDEALS AND $\mathcal{L}$ -FUZZY FILTERS IN RESIDUATED MULTILATTICE

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ABSTRACT: This document shows the work done to date. A literature review and our scientific contribution are presented just before giving perspectives for future works.

KEYWORDS: residuated lattice, multilattice, residuated multilattice, M-ideal, p-ideal, ideal, consistent ideal, congruence, homomorphism, fuzzy ideal, fuzzy prime ideal, prime fuzzy ideal, fuzzy maximal ideal and maximal fuzzy ideal.

1. Introduction

In this report, we will make an inventory of the work carried out within the framework of this thesis and give the perspectives envisaged to lead to the completion of this one.

We know that a computer is based on accurate data, for example to determine if a person is short or tall, it must not only know the accurate size of a person but also be in possession of an algorithm which divides inevitably a population in two distinct groups: tall and short. But it is easy for any one of us (human) to determine if a person is short or tall, and that without necessarily knowing his size. The idea of fuzzy logic is to transmit this world of the human reasoning (based on imprecise data or incomplete data) to a computer. Although in the mind of everybody the word "fuzzy" is of negative connotation, in reality it is not. The term "fuzzy" means indistinct, mixed or ill-defined.

The main areas of research and application of fuzzy logic are : automation (iron production, water purification,...), decision making, information processing, etc. It should be understood that ideals and filters play an important role in data processing because they allow information to be grouped into several classes with very specific characteristics, thus facilitating the stocking and processing of information. In addition, the notion of ideal was not introduced in residuated multilattices which are new algebraic structures defined by

Cabrera et al. in 2012. These structures generalize that of residuated lattice where, the notions of filters and ideals are known.

The application of fuzzy logic, the importance of ideals and filters in algebraic structures and the particularity of the residuated multilattice is our motivation for the choice of our research theme entitled : Ideals and $\mathcal{L}$ -fuzzy filters in residuated multilattice.

We have within the framework of this thesis proceed as follows : we started by define the notions of $\mathcal{L}$ -fuzzy ideals, $\mathcal{L}$ -fuzzy prime ideals and $\mathcal{L}$ -fuzzy maximal ideal of residuated lattice. We brought out the properties that they verify as well as the relations which link them. Thereafter, we are interested in the notion of $\mathcal{L}$ -fuzzy filters of residuated multilattices, where we have given examples and some properties. We also defined the concept of cosets of a $\mathcal{L}$ -fuzzy filter. We are now working on the concept of ideal and prime ideal of residuated multilattice. We will give the relations between this notion of ideal and the notions of ideal in multilattice and residuated lattice. Finally, we will study the notion of $\mathcal{L}$ -fuzzy ideals of residuated multilattice.

The rest of this article is made up of a literature review. It is followed by a part dealing with the various scientific contributions that we have made so far. Finally, this article will end with a conclusion and perspectives for future Work.

2. Literature review

Residuation is a fundamental concept of ordered structures. However, its interest as a research topic has been recently increased because of the fact that residuated lattices have been identified as the algebraic structures underlying sub structural logics, but it has application in different fields, such as network calculus, or in algebraic structures used in soft constraint satisfaction problems, or when considering the fuzzy extensions of crisp formalisms, for instance, description logic, or formal concept analysis. Current research conducted on residuated lattices shows them as suitable structures to represent roughness; other authors have focused on residuated frames as a valuable tool for solving both algebraic and logical problems; in another approach, residuation in a broad sense has been studied in relation with quantum structures and, furthermore, operations failing associativity, commutativity are used as underlying carrier to generalized residuated structures.

Several generalizations of lattices have been investigated in the literature for their applications, most precisely, in the area of fuzzy logic programming and coding theory. Some of these generalizations include hyperlattices [12], multilattices [4] and more recently residuated multilattices [3]. Multilattices, introduced by Benado [2] and contrariwise to bilattices, are defined on just one order relation, and it is the requirement of the existence of suprema and infima what is relaxed. Recently, Cordero et al [7] proposed an alternative algebraic definition of multilattice which is more closely related to that of lattice, allowing for natural definitions of related structures such that multisemilattices and, in addition, is better suited for applications; for instance, Medina et al developed a general approach to fuzzy logic programming based on a multilattice as underlying set of truth-values for the logic. A residuated multilattice is a partially ordered commutative residuated integral

monoid (a.k.a pocrim) whose poset is a multilattice [3]. In other words, residuated multilattices combine in a delicate manner the pocrim and multilattice structures on the same set. Therefore, residuated multilattices generalize both residuated lattices and multilattices.

Filters and ideals are very important on studying of logical algebras and the completeness of non-classical logics. In 2012, Cabrera et al. introduced the notion of filter in a multilattice and presented its relation to homomorphisms and congruences, then we move to residuated multilattices and consider the most adequate notion of filter, prove that the set of filters of a residuated multilattice is a complete lattice, and study again the relation between the filters, homomorphisms and congruences. So far, it should be noticed that the concept of ideal has not yet been introduced in residuated multilattice.

Dealing with certain and uncertain information is an important task of the artificial intelligence, in order to make computer simulate human being. To handle such information, in 1965, Zadeh [20] introduced the notion of fuzzy subset of a non empty set X as a function $\mu : X \rightarrow I$, where $I = [0; 1]$ is the unit interval of real numbers. Since then, many authors have been using the above original definition to setup fuzzy mathematical structures. The notion of fuzzy ideal has been studied in several algebraic structures such as rings [18], lattices [1, 13], MV-algebras [10], BL-algebras [15] and residuated lattices [11, 14, 17] and even in hyperstructures, for example in hyperlattices [12]. In 1988, Swamy [18], introduced the notion of prime fuzzy ideal and showed the difference with the notion of fuzzy prime ideal given by Attallah [1]. The study of fuzzy maximal ideals was done in rings [16], lattices [19] and MV-algebras [10], but this has not been done in residuated lattice. Many authors have investigated fuzzy algebraic notions taking the linearly ordered set $[0, 1]$ to be the set of degrees of membership. However, as Goguen [9] pointed out, in some situations, the structure of a complete bounded lattice L can be a suitable set of truth values. Tonga in [19], gives a new definition of a fuzzy set of a non-empty set by replacing the closed unit interval $[0, 1]$ by a complete bounded lattice. This new definition of fuzzy set was used by Kadji et al. [11] to study the notion of Fuzzy prime and maximal filters of residuated lattices.

3. Scientific contributions

In the case of this thesis, firstly, we have investigated the notion of $\mathcal{L}$ -fuzzy ideal of residuated lattice which generalizes what was done by Liu [14], by replacing the closed unit interval $[0; 1]$ by a complete bounded lattice $\mathcal{L}$. Besides, we have studied the notion of fuzzy prime ideal of residuated lattice and fuzzy prime ideal of the second kind and given their characterizations. The concept of prime fuzzy ideal was defined. Finally, we introduce the notions of maximal fuzzy ideal and fuzzy maximal ideal, it follows that a maximal fuzzy ideal is also a fuzzy maximal ideal, but the converse is not always verified.

Secondly, we have introduced the notion of $\mathcal{L}$ -fuzzy filter of residuated multilattice which generalizes the work of Maffeu et al, by replacing the closed unit interval $[0, 1]$ by a complete bounded lattice $\mathcal{L}$. Besides, we have studied the notion of $\mathcal{L}$ -fuzzy prime filter of

residuated multilattice and prime $\mathcal{L}$ -fuzzy filter, it appears that a prime $\mathcal{L}$ -fuzzy filter μ is a $\mathcal{L}$ -fuzzy prime if the 1-cut set of μ is a prime filter. The concepts of maximal $\mathcal{L}$ -fuzzy filter and $\mathcal{L}$ -fuzzy maximal filter was defined, it follows that a maximal $\mathcal{L}$ -fuzzy filter is also a $\mathcal{L}$ -fuzzy maximal filter if $\mathcal{L}$ is distributive, but the converse is not always verified. Finally, we introduce the notion of cosets of a $\mathcal{L}$ -fuzzy filter and we prove that the set of all cosets of a $\mathcal{L}$ -fuzzy filter is also a residuated multilattice.

Now, we are working on the notion of ideal of residuated multilattice. We have introduced the notion of ideal of residuated multilattice and we give some relations between this notion with congruences and homomorphisms. We also introduce the notion of consistent ideal and we prove that the quotient set of residuated multilattice with the ideal is a residuated lattice if and only if the ideal is a consistent ideal. In addition, we define the notions of prime ideal and prime ideal of second kind in residuated multilattice, we find out the relation between these two types of ideals.

4. Conclusion and perspectives

In this article, we have presented the various works already done in the framework of this thesis. We will finalize the work on the ideals and prime ideals of residuated multilattice and also investigate the notion of $\mathcal{L}$ -fuzzy ideals of residuated multilattice.

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