Intuitionistic Multi L-Fuzzy Subgroups and Intuitionistic Multi anti L-Fuzzy Subgroups

M Akilesh and C Gowrishankar

January 5, 2022

INTUITIONISTIC MULTI L-FUZZY SUBGROUPS AND INTUITIONISTIC MULTI ANTI L-FUZZY SUBGROUPS

Dr. M. AKILESH
Department of Mathematics, SRMV College of Arts and Science, Coimbatore-641 020.
Tamilnadu, India.
akileshmaths@gmail.com

C.GOWRISHANKAR
Department of Mathematics, Government Arts and Science College, Gudalur,
The Nilgiris – 643 212,Tamilnadu, India.
cgowrishankar88@gmail.com

ABSTRACT

In this paper, we define the algebraic structures of Intuitionistic Multi L–fuzzy subgroups and Intuitionistic Multi Anti L–fuzzy subgroups. The purpose of this study is to implement the fuzzy set theory and group theory in Intuitionistic Multi L–fuzzy subgroups and Intuitionistic Multi Anti L–fuzzy subgroups and also some related properties are investigated.

Keywords: L-Fuzzy subgroup, Intuitionistic L-fuzzy set (ILFS), Multi L–fuzzy subgroup (MLFSG), Anti L–fuzzy subgroup (ALFSG), Multi Anti L–fuzzy subgroup (MALFSG), Intuitionistic Multi L–fuzzy subgroup (IMLFSG), Intuitionistic Multi Anti L–fuzzy subgroup (IMALFSG).

I. INTRODUCTION

L.A.Zadeh [29] introduced the concept of fuzzy sets. A.Rosenfeld [21] applied this concept in group theory and semi group theory, and developed the theory of fuzzy subgroups. The concept of anti L–fuzzy subgroup was introduced by R.Biswas [7]. The concept of multi L–fuzzy subgroups was introduced by Sourier Sebastian and S.Babu Sundar [25]. The idea of Intuitionistic L–fuzzy set was given by K.T.Atanassov [5]. S.Sabu and T.V.Ramakrishnan [22] proposed the theory of multi L–fuzzy sets interms of multi L–dimensional membership functions.In all these studies, the closed unit interval [0,1] is taken as the membership Lattice.In this paper, we introduce the notion of Intuitionistic Multi L–fuzzy subgroups and Intuitionistic Multi Anti L–fuzzy subgroups of a group G and discuss some of its properties.

II. PRELIMINARIES

Throughout this paper G denotes an arbitrary multiplicative group with “e” is an identity element and L denotes an arbitrary Lattice with least element 0 and greatest element 1. The join and meet operations in L are denoted by max$(or \vee)$ and min$(or \wedge)$ respectively. A function $\mu: G \rightarrow L$ is called a multi L – fuzzy subset of G.

2.1 Definition

Let X be any non-empty set. A fuzzy set μ of X is $\mu: X \to [0,1]$.

2.2 Definition

Let $(G, .)$ be a group. A fuzzy subset μ of G is said to be L-fuzzy subgroup (LFSG) of G, if the following conditions are satisfied:

(i) $\mu(xy) \geq min\{\mu(x), \mu(y)\}$,
(ii) $\mu(x^{-1}) = \mu(x)$, for all $x, y \in G$.

2.3 Definition

A L-fuzzy subset μ of G is said to be anti L-fuzzy subgroup (ALFSG) of G, if the following conditions are satisfied:

(i) $\mu(xy) \leq max\{\mu(x), \mu(y)\}$,
(ii) $\mu(x^{-1}) = \mu(x)$, for all $x, y \in G$.

2.4 Definition

Let X be a fixed non-empty set. A Multi L-fuzzy subset (MLFS) μ in X is defined as a set of ordered sequences. $\mu = \{(x, \mu_1(x), \mu_2(x), \dots, \mu_i(x), \dots.): x \epsilon X\}$, where $\mu_i: X \to [0,1]$, for all i. Also note that, for all $i, \mu_i(x)$ is a decreasingly ordered sequence of elements.

ie., $\mu_1(x) \geq \mu_2(x) \geq \dots\dots \geq \mu_i(x) \geq \dots\dots$, for all $x \epsilon X$.

2.5 Definition

A Multi L-fuzzy subset μ of a group G is called a Multi L-fuzzy subgroup of G (MLFSG), if

(i) $\mu(x, y) \geq min\{\mu(x), \mu(y)\}$,
(ii) $\mu(x^{-1}) = \mu(x)$, for all $x, y \epsilon G$.

2.6 Definition

A Multi L-fuzzy subset μ of a group G is called a Multi Anti L-fuzzy subgroup of G (MALFSG), if

(i) $\mu(x, y) \leq max\{\mu(x), \mu(y)\}$,
(ii) $\mu(x^{-1}) = \mu(x)$, for all $x, y \epsilon G$.

2.7 Definition

Let X be a fixed non-empty set. An Intuitionistic L-fuzzy subset (ILFS) μ of X is an object of the form $\mu = \{x, \mu_A(x), \gamma_A(x): x \epsilon X\}$, where $\mu_A: X \to [0,1]$ and $\gamma_A: X \to [0,1]$.

Define the degree of membership and degree of non-membership of the element $x \epsilon X$ respectively with $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$, for all $x \epsilon X$.

Remark

(a) When $\mu_A(x) + \gamma_A(x) = 1$, ie.,when $\gamma_A(x) = 1 - \mu_A(x) = \mu_A^c(x)$. Then μ is called L-fuzzy set.

(b) We use the notation $\mu = (\mu_A, \gamma_A)$ to denote the Intuitionistic L-fuzzy subset (ILFS) μ of X.

III. INTUITIONISTIC MULTI L-FUZZY SUBGROUPS (IMLFSG)

3.1 Definition

Let X be a non-empty set. Let $\mu = \{x, \mu_A(x), \gamma_A(x): x \epsilon X\}$ in X is defined as a set of ordered sequences.

ie., $\mu = \{x, (\mu_{A_1}(x), \mu_{A_2}(x), \ldots., \mu_{A_i}(x), \ldots.), (\gamma_{A_1}(x), \gamma_{A_2}(x), \ldots., \gamma_{A_i}(x), \ldots.): x \epsilon X\}$.

Where $\mu_{A_i}: X \rightarrow [0,1]$, $\gamma_{A_i}: X \rightarrow [0,1]$ and $0 \leq \mu_{A_i}(x) + \gamma_{A_i}(x) \leq 1$ for all i.

Here, $\mu_1(x) \geq \mu_2(x) \geq . \ldots . \geq \mu_i(x) \geq . \ldots$, for all $x \epsilon X$ are decreasingly ordered sequence. Then the set μ is said to be an Intuitionistic Multi L-fuzzy subset (IMLFS) of X.

Remark

Since we arrange the membership sequence in decreasing order, the corresponding non-membership sequence may not be in decreasing or increasing order.

3.2 Definition

An Intuitionistic Multi L-fuzzy subset $\mu = \{x, \mu_A(x), \gamma_A(x): x \epsilon X\}$ of a group G is said to be Intuitionistic Multi L-fuzzy subgroup of G (IMLFSG) if it satisfies the following: For all $x, y \epsilon G$,

(i) $\mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\}$ and $\gamma_A(xy) \leq max\{\gamma_A(x), \gamma_A(y)\}$,
(ii) $\mu_A(x^{-1}) = \mu_A(x)$ and $\gamma_A(x^{-1}) = \gamma_A(x)$.

Or Equivalently μ is IMLFSG of G if and only if

$$\mu_A(xy^{-1}) \geq min\{\mu_A(x), \mu_A(y)\} \text{ and } \gamma_A(xy^{-1}) \leq max\{\gamma_A(x), \gamma_A(y)\}.$$

SOME BASIC OPERATIONS ON INTUITIONISTIC MULTI L-FUZZY SUBGROUPS

Let $A = \{x, \mu_A(x), \gamma_A(x): x \epsilon X\}$ and $B = \{x, \mu_B(x), \gamma_B(x): x \epsilon X\}$ be any two ILMFSG's of X, then

1. **Inclusion:** $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \epsilon X$.

2. Equality: $A = B \Leftrightarrow \mu_A(x) = \mu_B(x)$ and $\gamma_A(x) = \gamma_B(x)$ for all $x\epsilon X$.

3. Complement: $A^c = \{x, \gamma_A(x), \mu_A(x): x\epsilon X\}$

4. Union: $A \cup B = \{(x, (\mu_A \cup \mu_B)(x), (\gamma_A \cup \gamma_B)(x)): x\epsilon X\}$

Where, $(\mu_A \cup \mu_B)(x) = max(\mu_A(x), \mu_B(x))$ and $(\gamma_A \cup \gamma_B)(x) = min(\gamma_A(x), \gamma_B(x))$

5. Intersection: $A \cap B = \{(x, (\mu_A \cap \mu_B)(x), (\gamma_A \cap \gamma_B)(x)): x\epsilon X\}$

Where, $(\mu_A \cap \mu_B)(x) = min(\mu_A(x), \mu_B(x))$ and $(\gamma_A \cap \gamma_B)(x) = max(\gamma_A(x), \gamma_B(x))$

6. Addition: $A \oplus B = \{x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \gamma_A(x) + \gamma_B(x) - \gamma_A(x)\gamma_B(x): x\epsilon X\}$

7. Multiplication: $A \otimes B = \{(x, \mu_A(x)\mu_B(x), \gamma_A(x)\gamma_B(x): x\epsilon X)\}$

8. Difference: $A - B = \{x, \min(\mu_A(x), \gamma_B(x)), \max(\gamma_A(x), \mu_B(x)): x\epsilon X\}$

9. Symmetric Difference: $A \Delta B = \{(x, \max[\min(\mu_A(x), \gamma_B(x)), \min(\mu_B(x), \gamma_A(x))],$

$$\min\,[\max(\gamma_A(x), \mu_B(x), max(\gamma_B(x), \mu_A(x))]) : x\epsilon X\}$$

10. Cartesian Product: $A\text{x}B = \{(\mu_A(x)\mu_B(x), \gamma_A(x)\gamma_B(x)): x\epsilon X\}$

3.1 Theorem

Let μ be an Intuitionistic Multi L-fuzzy subgroup of a group G and e is the identity element of G. Then prove that

(i) $\mu_A(x) \leq \mu_A(e)$ and $\gamma_A(x) \geq \gamma_A(e)$, for all $x\epsilon G$.

(ii) The subset $H = \{x\epsilon G / \mu_A(x) = \mu_A(e)$ and $\gamma_A(x) = \gamma_A(e)\}$ is a subgroup of G.

Proof

(i) Let $x\epsilon G$

$$\begin{aligned} \mu_A(x) &= \min\{\mu_A(x), \mu_A(x)\} \\ &= \min\{\mu_A(x), \mu_A(x^{-1})\} \\ &\leq \mu_A(xx^{-1}) = \mu_A(e) \end{aligned}$$

Hence, $\mu_A(x) \leq \mu_A(e)$

And $\gamma_A(x) = \max\{\gamma_A(x), \gamma_A(x)\}$

$$= \max\{\gamma_A(x), \gamma_A(x^{-1})\}$$

$$\geq \gamma_A(xx^{-1}) = \gamma_A(e)$$

Hence, $\gamma_A(x) \geq \gamma_A(e)$

(ii) Let $H = \{x \epsilon G / \mu_A(x) = \mu_A(e) \text{ and } \gamma_A(x) = \gamma_A(e)\}$

Clearly, H is non-empty set as $e \epsilon H$. To prove that H is a subgroup of G.

Let $x, y \epsilon H$. Then $\mu_A(x) = \mu_A(y) = \mu_A(e)$

$$\mu_A(xy^{-1}) \geq \min\{\mu_A(x), \mu_A(y^{-1})\}$$

$$= \min\{\mu_A(x), \mu_A(y)\}$$

$$= \min\{\mu_A(e), \mu_A(e)\} = \mu_A(e)$$

Therefore, $\mu_A(xy^{-1}) \geq \mu_A(e)$ .... (1)

And obviously $\mu_A(e) \geq \min\{\mu_A(e), \mu_A(e)\}$

$$= \min\{\mu_A(x), \mu_A(y)\}$$

$$= \min\{\mu_A(x), \mu_A(y^{-1})\} \geq \mu_A(xy^{-1})$$

Therefore, $\mu_A(e) \geq \mu_A(xy^{-1})$ .... (2)

From (1) and (2) gives, $\mu_A(xy^{-1}) = \mu_A(e)$

Let $x, y \epsilon H$. Then $\gamma_A(x) = \gamma_A(y) = \gamma_A(e)$

$$\gamma_A(xy^{-1}) \leq \max\{\gamma_A(x), \gamma_A(y^{-1})\}$$

$$= \max\{\gamma_A(x), \gamma_A(y)\}$$

$$= \max\{\gamma_A(e), \gamma_A(e)\} = \gamma_A(e)$$

Therefore, $\gamma_A(xy^{-1}) \leq \gamma_A(e)$ .... (3)

And obviously $\gamma_A(e) \leq \max\{\gamma_A(e), \gamma_A(e)\}$

$$= \max\{\gamma_A(x), \gamma_A(y)\}$$

$$= \max\{\gamma_A(x), \gamma_A(y^{-1})\} \leq \gamma_A(xy^{-1})$$

Therefore, $\gamma_A(e) \leq \gamma_A(xy^{-1})$ .... (4)

From (3) and (4) gives, $\gamma_A(xy^{-1}) = \gamma_A(e)$ and $xy^{-1} \in H$.

Clearly, H is a subgroup of G.

3.2 Theorem

Prove that μ is an Intuitionistic L-fuzzy subgroup of G if and only if μ^c is an Intuitionistic Multi L-fuzzy subgroup of G.

Proof

Suppose μ is a ILFSG of G. Then for all $x, y \in G$.

Case (i) : The first property of ILFSG gives,

$$\mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\} \quad \Leftrightarrow \text{ Taking complement on both sides}$$

$$\Leftrightarrow \quad 1 - \mu_A(xy) \geq 1 - min\{1 - \mu_A(x), 1 - \mu_A(y)\}$$

$$\Leftrightarrow \quad \gamma_A(xy) \leq max\{\gamma_A(x), \gamma_A(y)\}$$

Case (ii) : The second property of ILFSG gives,

$$\mu_A(x) = \mu_A(x^{-1}) \quad \Leftrightarrow \text{ Taking complement on both sides}$$

$$\Leftrightarrow \quad 1 - \mu_A(x) = 1 - \mu_A(x^{-1})$$

$$\Leftrightarrow \quad \gamma_A(x) = \gamma_A(x^{-1})$$

In both the cases, $\mu_A(x) = 1 - \gamma_A(x)$ is a complement of μ $(ie., \mu^c)$ satiesfies the conditions for an IMLFSG.

Hence, μ^c is an IMLFSG of G.

3.3 Theorem

If μ is an Intuitionistic Multi L-fuzzy subgroup of G if and only if prove that

(i) $\mu_A(xy^{-1}) \geq \min\{\mu_A(x), \mu_A(y)\}$ and
(ii) $\gamma_A(xy^{-1}) \leq \max\{\gamma_A(x), \gamma_A(y)\}$

Proof

(i) Let μ be a IMLFSG of G for all $x, y \in G$, then

$$\Leftrightarrow \quad \mu_A(xy^{-1}) \geq \min\{\mu_A(x), \mu_A(y^{-1})\}$$

$$\Leftrightarrow \quad = \min\{\mu_A(x), \mu_A(y)\} \qquad [\because \text{By definition 3.2}]$$

$$\Leftrightarrow \quad \mu_A(xy^{-1}) \geq \min\{\mu_A(x), \mu_A(y)\}$$

(ii) Let μ be a IMLFSG of G for all $x, y \in G$, then

$$\Leftrightarrow \quad \gamma_A(xy^{-1}) \le \max\{\gamma_A(x), \gamma_A(y^{-1})\}$$

$$\Leftrightarrow \quad = \max\{\gamma_A(x), \gamma_A(y)\} \qquad [\because \text{By definition 3.2}]$$

$$\Leftrightarrow \quad \gamma_A(xy^{-1}) \le \max\{\gamma_A(x), \gamma_A(y)\}$$

Hence the proof.

3.4 Theorem

Let X be a non-empty set. For every two Intuitionistic Multi L-fuzzy subgroup of A and B in X, then prove that

(i) $(A \cup B) = A \cup B$ and (ii) $(A \cap B) = A \cap B$

Proof

Given X be a non-empty set.

For every two IMLFSG of A and B in X.

Then $A = \{x, \mu_A(x), \gamma_A(x): x\epsilon X\}$ and $B = \{x, \mu_B(x), \gamma_B(x): x\epsilon X\}$.

$$\begin{aligned} \text{(i)} \quad (A \cup B) &= \Big\{\Big(x, \big(\mu_A(x) \cup \mu_B(x)\big), \big(\gamma_A(x) \cup \gamma_B(x)\big)\Big) : x\epsilon X\Big\} \\ &= \{(x, (\mu_A \cup \mu_B)(x), (\gamma_A \cup \gamma_B)(x)) : x\epsilon X\} \\ &= \Big\{\Big(x, max\big(\mu_A(x), \mu_B(x)\big), min\big(\gamma_A(x), \gamma_B(x)\big)\Big) : x\epsilon X\Big\} = A \cup B \end{aligned}$$

$$\therefore \ (A \cup B) = A \cup B$$

$$\begin{aligned} \text{(ii)} \quad (A \cap B) &= \Big\{\Big(x, \big(\mu_A(x) \cap \mu_B(x)\big), \big(\gamma_A(x) \cap \gamma_B(x)\big)\Big) : x\epsilon X\Big\} \\ &= \{(x, (\mu_A \cap \mu_B)(x), (\gamma_A \cap \gamma_B)(x)) : x\epsilon X\} \\ &= \Big\{\Big(x, min\big(\mu_A(x), \mu_B(x)\big), max\big(\gamma_A(x), \gamma_B(x)\big)\Big) : x\epsilon X\Big\} = A \cap B \end{aligned}$$

$$\therefore \ (A \cap B) = A \cap B$$

Hence the theorem.

3.5 Theorem

Prove that μ is an Intuitionistic Multi L-fuzzy subgroup of G, then

$H = \{x\epsilon G / \mu_A(x) = 1, \gamma_A(x) = 0\}$ is either empty or is a subgroup of G.

Proof

Given μ is an IMLFSG of G.

To prove that $H = \{x\epsilon G / \mu_A(x) = 1, \gamma_A(x) = 0\}$ is either empty or is a subgroup of G.

Case (i) : If no element satisfies this condition, then H is empty.

Case (ii) : Suppose for all $x, y\epsilon H$.

By theorem 3.3, gives $\mu_A(xy^{-1}) \geq \min\{\mu_A(x), \mu_A(y^{-1})\}$

$= \min\{\mu_A(x), \mu_A(y)\}$ [$\because$ By definition 3.2]

$= \min\{1,1\} = 1$ [$\because \mu_A(x) = 1$]

$\therefore \mu_A(xy^{-1}) = 1$ [$\because \mu_A: X \rightarrow [0,1]$] .. …………… (1)

Again by theorem 3.3, $\gamma_A(xy^{-1}) \leq \max\{\gamma_A(x), \gamma_A(y^{-1})\}$

$= \max\{\gamma_A(x), \gamma_A(y)\}$ [$\because$ By definition 3.2]

$= \max\{0,0\} = 0$ [$\because \gamma_A(x) = 0$]

$\therefore \gamma_A(xy^{-1}) = 0$ [$\because \gamma_A: X \rightarrow [0,1]$] ……………… (2)

From (1) and (2) we get, $xy^{-1}\epsilon H$.

Therefore, H is a subgroup of G.

Hence H is either empty or is a subgroup of G.

3.6 Theorem

If μ is an Intuitionistic Multi L-fuzzy subgroup (IMLFSG) of a group G. For all $x, y, e \in G$ then prove that

(i) $\mu_A(xy^{-1}) = \mu_A(e)$ gives $\mu_A(x) = \mu_A(y)$.
(ii) $\gamma_A(xy^{-1}) = \gamma_A(e)$ gives $\gamma_A(x) = \gamma_A(y)$.

Proof

Let $x, y, e \in G$.

(i) Now, $\mu_A(x) = \mu_A(xy^{-1}y)$

$\geq \min\{\mu_A(xy^{-1}), \mu_A(y)\}$

$= \min\{\mu_A(e), \mu_A(y)\}$ [$\because \mu_A(xy^{-1}) = \mu_A(e)$]

$= \mu_A(ey) = \mu_A(y)$

$\therefore\ \mu_A(x) = \mu_A(y)$

(ii) Now, $\gamma_A(x) = \gamma_A(xy^{-1}y)$

$$\leq \max\{\gamma_A(xy^{-1}), \gamma_A(y)\}$$

$$= \max\{\gamma_A(e), \gamma_A(y)\} \qquad [\because \gamma_A(xy^{-1}) = \gamma_A(e)]$$

$$= \gamma_A(ey) = \gamma_A(y)$$

$\therefore\ \gamma_A(x) = \gamma_A(y)$

Hence the theorem.

IV. INTUITIONISTIC MULTI ANTI L-FUZZY SUBGROUPS (IMALFSG)

4.1 Definition

An Intuitionistic Multi L-fuzzy subset $\mu = \{x, \mu_A(x), \gamma_A(x): x\epsilon X\}$ of a group G is said to be Intuitionistic Multi Anti L-fuzzy subgroup of G (IMALFSG) if it satisfies the following:

For all $x, y\epsilon G$, then

(i) $\mu_A(xy) \leq max\{\mu_A(x), \mu_A(y)\}$ and $\gamma_A(xy) \geq min\{\gamma_A(x), \gamma_A(y)\}$,
(ii) $\mu_A(x^{-1}) = \mu_A(x)$ and $\gamma_A(x^{-1}) = \gamma_A(x)$.

Or Equivalently μ is IMALFSG of G if and only if

$$\mu_A(xy^{-1}) \leq max\{\mu_A(x), \mu_A(y)\} \text{ and } \gamma_A(xy^{-1}) \geq min\{\gamma_A(x), \gamma_A(y)\}.$$

4.1 Theorem

If μ is an Intuitionistic Multi Anti L-fuzzy subgroup of G if and only if prove that

(i) $\mu_A(xy^{-1}) \leq \max\{\mu_A(x), \mu_A(y)\}$ and
(ii) $\gamma_A(xy^{-1}) \geq \min\{\gamma_A(x), \gamma_A(y)\}$

Proof

(i) Let μ be a IMALFSG of G for all $x, y \in G$, then

$$\Leftrightarrow \mu_A(xy^{-1}) \leq \max\{\mu_A(x), \mu_A(y^{-1})\}$$

$$\Leftrightarrow \qquad = \max\{\mu_A(x), \mu_A(y)\} \qquad [\because \text{By definition 4.1}]$$

$$\Leftrightarrow \mu_A(xy^{-1}) \leq \max\{\mu_A(x), \mu_A(y)\}$$

(ii) Let μ be a IMALFSG of G for all $x, y \in G$, then

$\Leftrightarrow \quad \gamma_A(xy^{-1}) \geq \min\{\gamma_A(x), \gamma_A(y^{-1})\}$

$\Leftrightarrow \quad\quad\quad = \min\{\gamma_A(x), \gamma_A(y)\}$ [$\because$ By definition 4.1]

$\Leftrightarrow \quad \gamma_A(xy^{-1}) \geq \min\{\gamma_A(x), \gamma_A(y)\}$

Hence the proof.

4.2 Theorem

Let μ be an Intuitionistic Multi Anti L-fuzzy subset of a group $(G,.)$. If $\mu_A(e) = 0$ and $\gamma_A(e) = 1$ and $\mu_A(xy^{-1}) \leq \max\{\mu_A(x), \mu_A(y)\}$ and $\gamma_A(xy^{-1}) \geq \min\{\gamma_A(x), \gamma_A(y)\}$, for all $x, y \in G$, then μ is an Intuitionistic Multi Anti L-fuzzy subgroup (IMALFSG) of a group G.

Proof

Let $x, y \in G$ and e is the identity element in G. Also given $\mu_A(e) = 0$ and $\gamma_A(e) = 1$ and $\mu_A(xy^{-1}) \leq \max\{\mu_A(x), \mu_A(y)\}$ and $\gamma_A(xy^{-1}) \geq \min\{\gamma_A(x), \gamma_A(y)\}$.

To prove that μ is an IMALFSG of a group G.

(i) Now, $\mu_A(x^{-1}) = \mu_A(ex^{-1})$

$\leq \max\{\mu_A(e), \mu_A(x)\}$ [$\because \mu_A(xy^{-1}) \leq \max\{\mu_A(x), \mu_A(y)\}$]

$\leq \max\{0, \mu_A(x)\}$ [$\because \mu_A(e) = 0$]

$\therefore \mu_A(x^{-1}) = \mu_A(x)$

And $\gamma_A(x^{-1}) = \gamma_A(ex^{-1})$

$\geq \min\{\gamma_A(e), \gamma_A(x)\}$ [$\because \gamma_A(xy^{-1}) \geq \min\{\gamma_A(x), \gamma_A(y)\}$]

$\geq \min\{1, \gamma_A(x)\}$ [$\because \gamma_A(e) = 1$]

$\therefore \gamma_A(x^{-1}) = \gamma_A(x)$

(ii) Now, $\mu_A(xy) = \mu_A(x(y^{-1})^{-1})$

$\leq \max\{\mu_A(x), \mu_A(y^{-1})\}$ [$\because \mu_A(xy^{-1}) \leq \max\{\mu_A(x), \mu_A(y)\}$]

$\therefore \mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\}$ [$\because \mu_A(x) = \mu_A(x^{-1})$]

And $\gamma_A(xy) = \gamma_A(x(y^{-1})^{-1})$

$\geq \min\{\gamma_A(x), \gamma_A(y^{-1})\}$ [$\because \gamma_A(xy^{-1}) \geq \min\{\gamma_A(x), \gamma_A(y)\}$]

$\therefore \gamma_A(xy) \geq \min\{\gamma_A(x), \gamma_A(y)\}$ [$\because \gamma_A(x^{-1}) = \gamma_A(x)$]

Hence, μ is an IMALFSG of a group G.

4.3 Theorem

Let $(G,.)$ be a group. If μ is an Intuitionistic Multi Anti L-fuzzy subgroup of G, then prove that $\mu_A(xy) = \max\{\mu_A(x), \mu_A(y)\}$ and $\gamma_A(xy) = \min\{\gamma_A(x), \gamma_A(y)\}$ with $\mu_A(x) \neq \mu_A(y)$ and $\gamma_A(x) \neq \gamma_A(y)$, for each $x \text{ and } y \in G$.

Proof

Let $x, y \in G$ and μ be an IMALFSG of G. Also given that $\mu_A(x) \neq \mu_A(y)$ and $\gamma_A(x) \neq \gamma_A(y)$.

So without loss of generality assume that $\mu_A(x) < \mu_A(y)$ and $\gamma_A(x) > \gamma_A(y)$.

To prove that $\mu_A(xy) = \max\{\mu_A(x), \mu_A(y)\}$ and $\gamma_A(xy) = \min\{\gamma_A(x), \gamma_A(y)\}$.

(i) Now,

$$\begin{aligned}
\mu_A(y) &= \mu_A(x^{-1}xy) \\
&\leq \max\{\mu_A(x^{-1}), \mu_A(xy)\} \\
&= \max\{\mu_A(x), \mu_A(xy)\} && [\because \mu_A(x) = \mu_A(x^{-1})] \\
&= \mu_A(xy) && [\because \mu_A(x) < \mu_A(y)] \\
&\leq \max\{\mu_A(x), \mu_A(y)\} \\
&= \mu_A(y)
\end{aligned}$$

Therefore, $\mu_A(xy) = \mu_A(y) = \max\{\mu_A(x), \mu_A(y)\}$ for all $x, y \in G$.

(ii) Now,

$$\begin{aligned}
\gamma_A(y) &= \gamma_A(x^{-1}xy) \\
&\geq \min\{\gamma_A(x^{-1}), \gamma_A(xy)\} \\
&= \min\{\gamma_A(x), \gamma_A(xy)\} && [\because \gamma_A(x) = \gamma_A(x^{-1})] \\
&= \gamma_A(xy) && [\because \gamma_A(x) > \gamma_A(y)] \\
&\geq \min\{\gamma_A(x), \gamma_A(y)\} \\
&= \gamma_A(y)
\end{aligned}$$

Therefore, $\gamma_A(xy) = \gamma_A(y) = \min\{\gamma_A(x), \gamma_A(y)\}$ for all $x, y \in G$.

4.4 Theorem

If μ is an Intuitionistic Multi Anti L-fuzzy subgroup (IMALFSG) of a group G and if there is a sequence $\{x_n\}$ in G such that $\lim_{n\to\infty}[\max\{\,(\mu_A(x_n), \mu_A(x_n)\}] = 0$ and $\lim_{n\to\infty}[\min\{\,(\gamma_A(x_n), \gamma_A(x_n)\}] = 1$, then prove that $\mu_A(e) = 0$ and $\gamma_A(e) = 1$, where e is the identity element in G.

Proof

Let μ be an IMALFSG of a group G with e is the identity element in G and $x \in G$ be an arbitrary element.

We have $x \in G \Rightarrow x^{-1} \in G$ and hence $xx^{-1} = e$.

Given there is a sequence $\{x_n\} \in G$ such that $\lim_{n\to\infty}[\max\{(\mu_A(x_n), \mu_A(x_n)\}] = 0$ and $\lim_{n\to\infty}[\min\{(\gamma_A(x_n), \gamma_A(x_n)\}] = 1$.

To prove that $\mu_A(e) = 0$ and $\gamma_A(e) = 1$.

(i) Now, $\mu_A(e) = \mu_A(xx^{-1})$
$\leq \max\{\mu_A(x), \mu_A(x^{-1})\}$

$\therefore \mu_A(e) \leq \max\{\mu_A(x), \mu_A(x)\}$ $[\because \mu_A(x) = \mu_A(x^{-1})]$

For each n, we have $\mu_A(e) \leq \max\{\mu_A(x_n), \mu_A(x_n)\}$

Taking limit $n \to \infty$ on both sides we get,

$$\mu_A(e) \leq \lim_{n\to\infty}[\max\{(\mu_A(x_n), \mu_A(x_n)\}] = 0 \qquad [\because by\ given]$$

Therefore, $\mu_A(e) = 0$

(ii) Now, $\gamma_A(e) = \gamma_A(xx^{-1})$
$\geq \min\{\gamma_A(x), \gamma_A(x^{-1})\}$

$\therefore \gamma_A(e) \geq \min\{\gamma_A(x), \gamma_A(x)\}$ $[\because \gamma_A(x) = \gamma_A(x^{-1})]$

For each n, we have $\gamma_A(e) \geq \min\{\gamma_A(x_n), \gamma_A(x_n)\}$

Taking limit $n \to \infty$ on both sides we get,

$$\gamma_A(e) \geq \lim_{n\to\infty}[\min\{(\gamma_A(x_n), \gamma_A(x_n)\}] = 1 \qquad [\because by\ given]$$

Therefore, $\gamma_A(e) = 1$.

Hence the proof.

4.5 Theorem

If μ is an Intuitionistic Multi Anti L-fuzzy subgroup of G, then

(i) $\mu_A(xy) = \mu_A(yx)$ if and only if $\mu_A(x) = \mu_A(y^{-1}xy)$
(ii) $\gamma_A(xy) = \gamma_A(yx)$ if and only if $\gamma_A(x) = \gamma_A(y^{-1}xy)$ for all $x, y \in G$.

Proof

Let μ be a IMALFSG of G for all $x, y \in G$.

(i) Assume that $\mu_A(xy) = \mu_A(yx)$, we have

$$\mu_A(y^{-1}xy) = \mu_A(y^{-1}yx) = \mu_A(ex) = \mu_A(x).$$

$$\therefore \quad \mu_A(x) = \mu_A(y^{-1}xy)\ , \text{ for all } x, y \in G.$$

Conversely, assume that $\mu_A(x) = \mu_A(y^{-1}xy)$, we have

$$\mu_A(xy) = \mu_A(xyxx^{-1}) = \mu_A(yx).$$

$$\therefore \quad \mu_A(xy) = \mu_A(yx)\ , \text{ for all } x, y \in G.$$

Hence (i) is proved.

(ii) Also assume that $\gamma_A(xy) = \gamma_A(yx)$, we have

$$\gamma_A(y^{-1}xy) = \gamma_A(y^{-1}yx) = \gamma_A(ex) = \gamma_A(x).$$

$$\therefore \quad \gamma_A(x) = \gamma_A(y^{-1}xy)\ , \text{ for all } x, y \in G.$$

Conversely, assume that $\gamma_A(x) = \gamma_A(y^{-1}xy)$, we have

$$\gamma_A(xy) = \gamma_A(xyxx^{-1}) = \gamma_A(yx).$$

$$\therefore \ \gamma_A(xy) = \gamma_A(yx)\ , \text{ for all } x, y \in G.$$

Hence (ii) is proved.

4.6 Theorem

If A and B are Intuitionistic Multi Anti L-fuzzy subgroups of G and H, respectively, then AxB is an Intuitionistic Multi Anti L-fuzzy subgroup of GxH.

Proof

Let A and B be Intuitionistic Multi Anti L-fuzzy subgroups of G and H, respectively. Let x_1 and x_2 be in G, y_1 and y_2 be in H.

Then (x_1, y_1) and (x_2, y_2) are in GxH.

Now, $\mu_{A\text{x}B}[(x_1, y_1)(x_2, y_2)] = \mu_{A\text{x}B}((x_1x_2, y_1y_2)$

$$= \max\{\mu_A((x_1x_2), \mu_B(y_1y_2)\}$$

$$\leq \max[\max\{\mu_A(x_1), \mu_A(x_2)\}, \max\{\mu_B(y_1), \mu_B(y_2)\}]$$

$$= \max\left[\max\{\mu_A(x_1), \mu_B(y_1)\}, \max\{\mu_A(x_2), \mu_B(y_2)\}\right]$$

$$= \max\{\mu_{AxB}(x_1, y_1), \mu_{AxB}(x_2, y_2)\}$$

$\therefore \quad \mu_{AxB}[(x_1, y_1)(x_2, y_2)] \leq \max\{\mu_{AxB}(x_1, y_1), \mu_{AxB}(x_2, y_2)\}, \quad \forall\, x_1, x_2 \in G, \; y_1, y_2 \in H.$

And, $\gamma_{AxB}[(x_1, y_1)(x_2, y_2)] = \gamma_{AxB}((x_1x_2, y_1y_2)$

$$= \min\{\gamma_A((x_1x_2), \gamma_B(y_1y_2)\}$$

$$\geq \min\left[\min\{\gamma_A(x_1), \gamma_A(x_2)\}, \min\{\gamma_B(y_1), \gamma_B(y_2)\}\right]$$

$$= \min\left[\min\{\gamma_A(x_1), \gamma_B(y_1)\}, \min\{\gamma_A(x_2), \gamma_B(y_2)\}\right]$$

$$= \min\{\gamma_{AxB}(x_1, y_1), \gamma_{AxB}(x_2, y_2)\}$$

$\therefore \quad \gamma_{AxB}[(x_1, y_1)(x_2, y_2)] \geq \min\{\gamma_{AxB}(x_1, y_1), \gamma_{AxB}(x_2, y_2)\}, \quad \forall\, x_1, x_2 \in G, \; y_1, y_2 \in H.$

Hence AxB is an IMALFSG of GxH.

Hence the proof.

V. CONCLUSION

In this paper, we have seen some basic definitions related on Intuitionistic L-fuzzy subgroup (ILFSG) and Intuitionistic Anti L-fuzzy subgroup (IALFSG). Next we define Intuitionistic Multi L-fuzzy subgroup (IMLFSG) and Intuitionistic Multi Anti L-fuzzy subgroup (IMALFSG) with the help of ILFSG and IALFSG. Also we have investigated some properties and theorems based on IMLFSG and IMALFSG. Further work is in progress in order to develop the IMLFSG and IMALFSG.

REFERENCES

[1] Akgul.M, "Some Properties of Fuzzy Groups", J. Math. Anal. Appl. 133 (1988), 93-100.

[2] Akilesh.M and Muthuraj.R, "Some Properties of Multi L – Fuzzy Cosets, International Journal of Mathematical Archive, Vol.9 (5), 211 – 215, 2018.

[3] Akilesh.M, Senthil Kumar.A and Muthuraj.R, "Anti Product of Multi Anti L-Fuzzy Subgroup of a Group", International Journal of Pure and Applied Mathematics, Vol.117, No.20 (2017), 51-62.

[4] Anthony.J.M, Sherwood.H, "A Characterization of Fuzzy Subgroups", Fuzzy Sets and systems, 7 (1982), 297-305.

[5] Atanassov.K.T," Intuitionistic Fuzzy Sets", Fuzzy Sets & Systems 20(1986), No.1, 87- 96.

[6] Balasubramaniyan.K.R and Revathy.R, “Some Theorems on Intuitionistic Multi L-Fuzzy Subgroups”, International Journal of Scientific and Research Publications, Vol.10, Issue 1 (2020), 653-655.

[7] Biswas.R, “Fuzzy Subgroups And Anti Fuzzy Subgroups”, Fuzzy Sets and Systems 35 (1990), 121–124.

[8] Das.P.S, “Fuzzy Groups and Level Subgroups”, Journal of Mathematical Analysis and Applications, 84 (1981), 264-269.

[9] Ejegwa.P.A, Akubo.A.J and Joshua.O.M, “Intuitionistic Fuzzy Set And Its Application in Career Determination Via Normalized Euclidean Distance Method”, European Scientific Journal, Vol.10, No.15(2014), 529-536.

[10] Goguen.J.A,” L – Fuzzy Sets”, Journal of Mathematical Analysis and Applications, 18 (1967), 145–174.

[11] Liu.W.J, “Fuzzy Invariant Subgroup and Fuzzy Ideal”, Fuzzy Sets and Systems, 8 (1981), 133-139.

[12] Mordeson.J.N and Malik.D.S, “Fuzzy Commutative Algebra”, World Scientific Publishing Co. Pvt. Ltd., (1998).

[13] Mukherjee.T.K and Sen.M.K, “On Fuzzy Ideals of a Ring”, Fuzzy Sets and Systems, 21(1987), 99-104.

[14] Mukharjee.N.P and Bhattacharya.P, “Fuzzy Normal Subgroups and Fuzzy Cosets, Information Sciences, 34 (1984), 225-239.

[15] Mustafa Akgul, “Some properties of fuzzy groups”, Journal of mathematical Analysis and applications, 133, 93-100 (1988).

[16] Muthuraj.R and Balamurugan.S, “A Study on Intuitionistic Multi-Anti Fuzzy Subgroups”, Applied Mathematics and Sciences: An International Journal(MathSJ), Vol.1, No.2(2014), 35-52.

[17] Muthuraj.R and Malar Selvi.C, “Application of Intuitionistic Multi-Fuzzy Set in Evaluating a Efficient Basket Ball Player”, Journal of Applied Science and Computation, Vol.6, Issue III, March 2019.

[18] Muthuraj.R and Malar Selvi.C, “A Characterization Theorem for Multi-fuzzy Prime Ideal”, SSRG International Journal of Mathematics Trends and Technology, ISSN:2231-5373, 12–14.

[19] Palaniappan.N, Naganathan.S and Arjunan.K, "A Study on Intuitionistic L-Fuzzy Subgroups", Applied Mathematical Sciences, Vol.3, No.53(2009), 2619-2624.

[20] Pandiammal.P, "A Study on Intuitionistic Anti L-Fuzzy M-Subgroups", International Journal of Computer & Organization Trends, Vol.4, Issue 1(2014), 46-53.

[21] Rosenfeld.A, "Fuzzy Groups", Journal of Mathematical Analysis and Applications, 35 (1971), 512-517.

[22] Sabu.S and Ramakrishnan.T.V, "Multi-Fuzzy Sets", International Mathematical Forum, 50 (2010), 2471-2476.

[23] Satya Saibaba.G.S.V: "Fuzzy Lattice Ordered Groups", Southeast Asian Bulletin of Mathematics, 32, 749-766(2008).

[24] Sharma.P.K, "Intuitionistic Fuzzy Groups", International Journal of Data Warehousing & Mining, Vol.1, Issue 1(2011), 86-94.

[25] Souriar Sebastian and Babu Sundar.S , "Commutative L-Fuzzy Subgroups Fuzzy Sets and Systems", 68 (1994), 115-121.

[26] Sunderrajan.K, Suresh.M and Muthuraj.R, "Commutative Multi L-Fuzzy Subgroups", International Journal of Engineering Research & Technology, Vol.2, Issue 4 (2013), 1143-1159.

[27] Wang.G.J, "Order-Homomorphism on Fuzzes", Fuzzy Sets and Systems, 12 (1984), 281- 288.

[28] Ying-Ming.L, "Structures of Fuzzy Order Homomorphisms", Fuzzy Sets and Systems, 21 (1987), 43-51.

[29] Zadeh.L.A, "Fuzzy Sets", Information and Control, 8 (1965), 338-353.